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Flat Directions in Z_{2n} Orbifold Models

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Abstract

We study generic features related to matter contents and flat directions in Z_{2n} orbifold models. It is shown that Z_{2n} orbifold models have massless conjugate pairs, R and $\overline{R}$, in certain twisted sectors as well as one of untwisted subsectors. Using these twisted sectors, Z_{2n} orbifold models are classified into two types. Conjugate pairs, R and $\overline{R}$, lead to D-flatness as $\langle R\rangle = \langle\overline{R}\rangle \neq 0$. We investigate generic superpotentials derived from orbifold models so as to show that this direction is indeed a flat direction.

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1 Introduction

Superstring theory is a promising candidate for the unified theory of all interactions including gravity. A number of 4-dimensional string models have been constructed through several types of constructions, e.g., Calabi-Yau construction [1], orbifold construction [2] and fermionic construction [3]. Although some interesting attempts have been done, completely realistic models have not been found yet.

For example, it is shown that Z_N orbifold models cannot have the standard model gauge group $SU(3) \times SU(2) \times U(1)$ without nontrivial Wilson lines [4]. The introduction of Wilson lines can lead to models with smaller gauge groups including the standard model gauge group and reduced matter multiplets [5]. However, these models have, in general, several extra $U(1)$ symmetries and many extra matter fields besides those in the minimal supersymmetric standard model. If the effective field theories derived from string models have flat directions, gauge groups can break into smaller groups and extra matter fields can become massive. Thus study on flat directions in those models is important from the viewpoint of realistic model constructions [6]. Actually some semirealistic models have been obtained through flat directions in orbifold models [7]. Further these flat directions could relate different models in string vacua.

Recently much interest is paid to understand classical and quantum moduli spaces (flat directions) of models with $N \geq 1$ supersymmetry (SUSY) [8] and string models with $N \geq 1$ and $d \geq 4$ [9]. Such a study leads to the understanding of some nonperturbative aspects like dualities between SUSY models and string dualities.

In this way, study on flat directions is more and more important in supersymmetric theories from several aspects. In the effective theories from Z_N orbifold models, flat directions have been investigated chiefly for Z_3 orbifold models as well as $Z_3 \times Z_3$ orbifold models [6, 7, 10]. The other Z_{2n} orbifold models have been little studied though they have different features from Z_3 orbifold models. For example, (2,2) Z_{2n} orbifold models have $\overline{27}$ massless matter fields of the E_6 group as well as 27 massless matter fields, while the (2,2) Z_3 orbifold model as well as the (2,2) $Z_3 \times Z_3$ orbifold model has only 27 massless matter fields, but not $\overline{27}$ fields. Thus in the Z_{2n} orbifold models, we have candidates for flat directions as $\langle 27 \rangle = \langle \overline{27} \rangle \neq 0$. In general, (0,2) Z_{2n} orbifold models also have other conjugate pairs, R and $\overline{R}$, and their VEVs could lead to a large symmetry breaking. Such models should be paid attention to because they might lead to a realistic model through breakings by flat directions. If the models have generic features related to flat directions, we have to take into account them for a realistic model building from the beginning.

Conjugate massless pairs, R and $\overline{R}$, appear not only in Z_{2n} orbifold models but also in 4-dimensional string models by other constructions, e.g. Calabi-Yau compactifications and fermionic constructions. Thus study on flat directions,

$\langle R \rangle = \langle \overline{R} \rangle \neq 0$, in Z_{2n} orbifold models is also interesting as an example for generic four-dimensional string models.

In this paper, we study generic features related to matter contents and flat directions in Z_{2n} orbifold models. It is shown that Z_{2n} orbifold models have massless conjugate pairs, R and $\overline{R}$, in certain twisted sectors as well as one of untwisted subsectors. Using these twisted sectors, Z_{2n} orbifold models are classified into two types. This classification is very useful to study flat directions and the breaking mechanism by them in Z_{2n} orbifold models. Conjugate pairs, R and $\overline{R}$, lead to D-flatness as $\langle R \rangle = \langle \overline{R} \rangle \neq 0$. We investigate generic superpotentials derived from orbifold models so as to show that vacuum expectation values (VEVs) of those pairs also lead generally to flat directions.

This paper is organized as follows. In the next section, massless spectra in Z_{2n} orbifold models are reviewed. Also selection rules for couplings in orbifold models are reviewed. In section 3, we study how conjugate pairs R and $\overline{R}$ appear in massless spectra of orbifold models. In subsection 3.1, it is shown that such pairs always appear in specific twisted and untwisted subsectors of Z_{2n} orbifold models. In terms of these sectors, we classify E_8 shifts to construct Z_{2n} orbifold models into three classes in subsection 3.2, and further using combinations of E_8 shifts and E'_8 shifts and twisted sectors with conjugate pairs, Z_{2n} orbifold models are classified into only two types in subsection 3.3. In subsection 4.1, it is shown VEVs of these conjugate pairs can lead to flat directions. These are generic flat directions which all Z_{2n} orbifold models have. In order to show the existence of flat directions, we discuss selection rules for renormalizable and nonrenormalizable couplings. In subsection 4.2, we study an explicit model to ascertain the results in subsection 4.1. We show that explicit models have flat directions other than generic flat directions. Also it is shown how many massless matter fields gain masses along flat directions. In subsection 4.3, the situation of flat directions in models with nontrivial Wilson lines is discussed. Wilson lines resolve degeneracy of massless matter fields, but similar flat directions can be found. In subsection 4.4, comments related to anomalies are given. Section 5 is devoted to conclusions and discussions. In Appendix A, structures of Z_{2n} orbifold models are summarized.

2 Orbifold Models

2.1 Massless spectrum

The $E_8 \times E'_8$ heterotic string theory consists of a bosonic string in the (4+6)-dimensional space-time, its right-moving superpartner and a left-moving gauge part, whose momenta P^I ($I = 1 \sim 16$) span an $E_8 \times E'_8$ lattice $\Gamma_{E_8 \times E'_8}$. Nonzero E_8 root vectors are represented as

$$(\underline{\pm 1, \pm 1, 0, 0, 0, 0, 0, 0}), \tag{1}$$

$$(\underline{\pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}}), \tag{2}$$

where the underline denotes any possible permutation of elements and the number of minus signs in Eq. (2) should be even. It is obvious that if P^I belongs to Eq.(1) or (2), $-P^I$ also belongs. The right-moving fermionic string is bosonized into the bosonic string whose momenta p^t $(t = 0 \sim 4)$ span an $SO(10)$ weight lattice $\Gamma_{SO(10)}$. Vector and spinor weights correspond to bosonic and fermionic fields, respectively.

In orbifold models [2, 11, 4, 12], the 6-dimensional space is compactified into an orbifold, which is a division of a 6-dimensional torus T^6 by its automorphism (twist) θ. Z_N orbifold models satisfy $\theta^N = 1$ *. To preserve the world-sheet supersymmetry and the modular invariance, this twist θ should be associated with shifts on $\Gamma_{SO(10)}$ and $\Gamma_{E_8 \times E_8'}$, v^t and V^I. Here the elements corresponding to the 4-dimensional space-time, v^t $(t = 0, 4)$, vanish and the other elements are related with eigenvalues of θ as $\theta = \mathrm{diag}\exp[2\pi i v^i]$ $(i = 1, 2, 3)$. For the shift V^I, the consistency with $\theta^N = 1$ leads to the constraint that NV^I should be on $\Gamma_{E_8 \times E_8'}$. This number N is called the order of this shift V^I. Further the modular invariance requires the following relation between V^I and v^i:

$$N \sum_{i=1}^{3} (v^i)^2 - N \sum_{I=1}^{16} (V^I)^2 = \text{even}. \tag{3}$$

The twists leading to the $N = 1$ space-time supersymmetry in $d = 4$ are classified into nine combinations of eigenvalues of θ, i.e., Z_3, Z_4, Z_6-I, Z_6-II, Z_7, Z_8-I, Z_8-II, Z_{12}-I and Z_{12}-II orbifold models. The corresponding values of v^i are shown in Appendix A except Z_3 and Z_7 orbifold models. These twists can be realized as Coxeter elements on Lie lattices Γ_6, which are used to construct T^6 as $T^6 = R^6/\Gamma_6$. The Coxeter element is a product of all Weyl reflections corresponding to the Lie lattice. For example, the Z_4 orbifold has $v^i = 1/4(1, 1, -2)$. The corresponding twist θ is obtained as the Coxeter element of an $SO(5)^2 \times SU(2)^2$ lattice. We denote simple roots by e_a $(a = 1 \sim 6)$, where one of the $SO(5)$ lattices is spanned by (e_1, e_2) or (e_3, e_4) and one of the $SU(2)$ lattices is spanned by e_5 or e_6. These vectors are transformed under θ as

$$\theta e_a = e_a + 2e_{a+1}, \quad \theta e_{a+1} = -e_a - e_{a+1}, \qquad (a = 1, 3), \tag{4}$$

$$\theta e_a = -e_a, \qquad (a = 5, 6). \tag{5}$$

It is easy to show this twist θ has eigenvalues $\exp[2\pi i/4(1, 1, -2)]$. Coxeter elements of other lattices can realize these eigenvalues. For example, the Coxeter elements of the $SO(5) \times SU(2) \times SU(4)$ and $SU(4)^2$ lattices have the same eigenvalues. We can obtain the Z_4 orbifold as a division of the $SO(5)^2 \times SU(2)^2$ torus by this twist θ (4) and (5). Similarly we can construct other Z_N orbifolds.

* We have another type of orbifold models, which have two independent twists and are called $Z_N \times Z_M$ orbifold models [13].

There are two types of closed strings on orbifolds. One is closed on tori and is called an untwisted string. The other is a twisted string. For the θ^k-twisted sector T_k, the string coordinate satisfies the following boundary condition:

$$x^i(\sigma = 2\pi) = (\theta^k x)^i(\sigma = 0) + e^i, \tag{6}$$

where e^i is a lattice vector. Zero-modes of the twisted string satisfy the same condition as Eq. (6). These zero-modes are called fixed points and denoted by corresponding space group elements (θ^k, e^i). For example, the θ-twisted sector of the above Z_4 orbifold has 16 fixed points as

$$(\theta, ie_2 + je_4 + ke_5 + \ell e_6), \tag{7}$$

where $i, j, k, \ell = 0, 1$.

Further the θ^2-twisted sector has 16 fixed points because each $SO(5)$ plane has four fixed points as

$$(\theta^2, ie_a + je_{a+1}), \tag{8}$$

where $i, j = 0, 1$ and $a = 1, 3$. Note that the $e_5 - e_6$ plane is completely fixed under θ^2. Among Eq. (8), the fixed points (θ^2, ie_a) are also fixed under θ. However, the others are transformed under θ as

$$(\theta^2, e_{a+1}) \longleftrightarrow (\theta^2, e_a + e_{a+1}), \tag{9}$$

up to their conjugacy class. Physical states should be eigenstates of θ. Hence we have to take linear combinations of the states corresponding to the above fixed points as [14, 12]

$$|\theta^2, e_{a+1}\rangle \pm |\theta^2, e_a + e_{a+1}\rangle. \tag{10}$$

These states have eigenvalues ± 1, respectively. In addition, we have two eigenstates, $|\theta^2, ie_a\rangle$, whose eigenvalue is 1. We construct ground states for the string on the 6-dimensional orbifold as tensor products of these states corresponding to two $SO(5)$ planes. Then we have 10 states with the eigenvalue $+1$ and 6 states with the eigenvalue -1, as explicitly shown in Appendix A. The same structure on fixed points is obtained from the different lattices, $SO(5) \times SU(2) \times SU(4)$ and $SU(4)^2$.

In general, the θ^k-twisted sector of the Z_N orbifold model has the following θ-eigenstates:

$$|\theta^k, e^i\rangle + e^{-i\gamma}|\theta^k, \theta e^i\rangle + \cdots + e^{-i\gamma(m-1)}|\theta^k, \theta^{m-1} e^i\rangle, \tag{11}$$

where (θ^k, e^i) is a fixed point of T_k and m is the least number so that (θ^k, e^i) is fixed under θ^m. This state has an eigenvalue $e^{i\gamma}$ where $\gamma = 2\pi\ell/m$ with $\ell =$

integer. The T_n sectors of Z_{2n} orbifold models have the same 16 fixed points as those of T_2 in Z_4 orbifold models, i.e.,

$$(\theta^n, ie_1 + je_2 + ke_3 + \ell e_4), \tag{12}$$

where $i, j, k, \ell = 0, 1$. For the T_n sectors of Z_{2n} orbifold models, θ-eigenstates are written explicitly in Appendix A

For the T_k sector, the $SO(10)$ and $E_8 \times E_8'$ momenta are shifted as $p^t + kv^t$ and $P^I + kV^I$. Independent shifts V^I on the E_8 lattice for each Z_N orbifold are classified in Ref. [15]. It is convenient to describe mass formulae in the light-cone gauge, where $SO(10)$ momenta are reduced into transverse $SO(8)$ momenta $p^t + kv^t$ ($t = 1 \sim 4$). The massless conditions are written as

$$\frac{1}{2}\sum_{t=1}^{4}(p^t + kv^t)^2 + N_R^{(k)} + c_k - \frac{1}{2} = 0, \tag{13}$$

for the right-moving T_k sector, and

$$\frac{1}{2}\sum_{I=1}^{16}(P^I + kV^I)^2 + N_L^{(k)} + N_{E_8\times E_8'} + c_k - 1 = 0, \tag{14}$$

for the left-moving T_k sector, where

$$c_k \equiv \frac{1}{2}\sum_{i=1}^{3}\eta^i_{(k)}(1-\eta^i_{(k)}), \quad \eta^i_{(k)} \equiv |kv^i| - \mathrm{Int}|kv^i|, \tag{15}$$

and $N_L^{(k)}$ ($N_R^{(k)}$) denotes the left-moving (right-moving) oscillator number. Here the T_k sector with $k = 0$ corresponds to the untwisted sector U. In such a case, we should consider an additional contribution from the 6-dimensional internal space momenta to the massless conditions. Further $N_{E_8\times E_8'}$ denotes the oscillator number for the gauge part and this contributes only for the untwisted sector. For the untwisted sector, bosonic fields have SO(8) momenta $p^t = (\underline{1,0,0,0})$. For twisted sectors, the SO(8) momenta satisfying Eq. (13) are shown in Refs. [14, 12]. For example, T_n massless bosonic states of Z_{2n} orbifold models have $c_n = 1/4$ and $p^t + nv^t = 1/2(1,1,0,0)$ in the base where $nv^t = 1/2(1,1,0,0)$ (mod Z) as Appendix A. Eq. (14) for the untwisted sector ($k = c_k = 0$) is satisfied by the states with $\sum_I (P^I)^2 = 2$, i.e., Eqs. (1) and (2) as well as the states with $N_{E_8\times E_8'} = 1$ and vanishing momenta.

Physical states should be invariant under the full Z_N transformation, i.e., transformations of the $SO(8)$ and $E_8 \times E_8'$ lattices by shifts v^t and V^I and the θ-twist on the 6-dimensional orbifold. For the untwisted sector, this Z_N invariance leads to the following constraint on their momenta:

$$\sum_{I=1}^{16} P^I V^I - \sum_{i=1}^{3} p^i v^i = \text{integer}. \tag{16}$$

The untwisted states with $p^t = (0,0,0,1)$, i.e. $\sum P^I V^I$ =integer, correspond to unbroken gauge bosons and the untwisted states with $p^t = (\underline{1,0,0},0)$ correspond to bosonic massless matter fields. We denote these three untwisted subsectors corresponding to $(1,0,0,0)$, $(0,1,0,0)$ and $(0,0,1,0)$ by U_1, U_2 and U_3, respectively.

The T_k state with momenta $p^t + kv^t$ and $P^I + kV^I$ has an eigenvalue for the full Z_N transformation as

$$\Delta_k = O^{(k)} e^{i\gamma} \tilde{\Delta}_k, \tag{17}$$

$$\begin{aligned} \tilde{\Delta}_k &= \exp[2\pi i(\frac{k}{2}\,(\sum_i (v^i)^2 - \sum_I (V^I)^2) \\ &\quad + \sum_I (P^I + kV^I)V^I - \sum_i (p^i + kv^i)v^i], \end{aligned} \tag{18}$$

where γ is the contribution from the ground state for the 6-dimensional orbifold like Eq. (11) and $\tilde{\Delta}_k$ is a contribution of the $SO(8)$ and $E_8 \times E_8'$ parts. Further $O^{(k)}$ denotes a contribution from oscillators. We call Δ_k the (generalized) GSO phase [11, 16]. Physical states should have $\Delta_k = 1$, that is, the 6-dimensional ground states with $e^{i\gamma} = (O^{(k)}\tilde{\Delta}_k)^{-1}$ are selected as physical states.

2.2 Selection rules for couplings

Here we review selection rules for couplings [17] including nonrenormalizable couplings [18, 10] †. For bosonic states in the non-oscillated T_k sector, vertex operators are obtained in the -1-picture as

$$V_{-1} = e^{-\phi} e^{iKx} e^{i(p+kv)H} e^{i(P+kV)x} \sigma_{f\gamma}, \tag{19}$$

where ϕ and H denote the bosonized superconformal ghost and the bosonized right-moving fermionic string, and e^{iKx} and $e^{i(P+kV)x}$ are the 4-dimensional part and the gauge part. Here $\sigma_{f\gamma}$ is the twisted field associated with the fixed point f and the eigenvalue $e^{i\gamma}$. Also vertex operators of fermionic states in the $-1/2$-picture are written as

$$V_{-1/2} = e^{-\phi/2} e^{iKx} e^{i(p+kv)_f H} e^{i(P+kV)x} \sigma_{f\gamma}, \tag{20}$$

where the momenta $(p+kv)^i_f$ for fermionic states are obtained by those for bosonic states as $(p+kv)^i_f = (p+kv)^i - 1/2$. These vertex operators are related by the supersymmetry. For the untwisted sector, we do not need $\sigma_{f\gamma}$. We can change the picture by the picture changing operator [20], which includes

$$e^{\phi} e^{-i\alpha_i H} \partial X_i, \tag{21}$$

†See also Ref.[19].

where α_1, α_2 and α_3 are (1,0,0), (0,1,0) and (0,0,1), and ∂X_i is an oscillator for the i-th plane. Note that if we change pictures, oscillators are included even in the vertex operators (19) and (20), which correspond to the massless states with $N_L^{(k)} = 0$ in Eq.(14). Similarly we can write vertex operators corresponding to the massless states with $N_L^{(k)} \neq 0$ in Eq.(14).

Let us study $V_F V_F V_B^\ell$ couplings, where V_F and V_B are vertex operators for fermionic and bosonic states, respectively. These vertex operators consist of several parts as Eqs. (19) and (20). Each of them provides selection rules for nonvanishing couplings. In nonvanishing couplings, a product of space group elements (θ^k, e^i) corresponding to f should be equivalent to (1,0) up to their conjugacy class and a product of $e^{i\gamma}$ should be unity. For 16 fixed points in T_n sectors of Z_{2n} orbifold models (12), couplings are allowed if a product of the corresponding space group elements satisfies $\prod(\theta^n, i_a e_1 + j_b e_2 + k_c e_3 + \ell_d e_4) = (1, 0)$, i.e..

$$\sum_a i_a = \text{even}, \quad \sum_b j_b = \text{even}, \quad \sum_c k_c = \text{even}, \quad \sum_d \ell_d = \text{even}. \tag{22}$$

Further $SO(8)$ momenta as well as $E_8 \times E_8'$ momenta should be conserved. In Refs.[14, 12], renormalizable couplings satisfying $SO(8)$ momentum conservation are shown explicitly. To match the background ϕ-charge, the sum of ϕ-charges for vertex operators should be -2. Moreover nonvanishing couplings should be invariant under the following Z_N transformation of oscillators;

$$\partial X_{i(k)} \longrightarrow e^{2\pi i k v^i} \partial X_{i(k)}, \tag{23}$$

where $\partial X_{i(k)}$ denotes the oscillator of T_k for the i-th plane.

For auxiliary fields of chiral fields, corresponding vertex operators in the 0-picture, V_A, are obtained from $V_{-1/2}$ through the supersymmetry [21]. Thus $V_A V_B V_B^\ell$ couplings have the same ϕ-charge and the same $SO(8)$ momenta as $V_F V_F V_B^\ell$ couplings. We can derive the same selection rules for $V_A V_B V_B^\ell$ couplings as $V_F V_F V_B^\ell$ couplings.

3 Classification of Z_{2n} orbifold models

3.1 Conjugate pairs in $\hat{U}_3$ and T_n of Z_{2n} orbifold models

In this subsection, we show that $\hat{U}_3$ and T_n sectors of Z_{2n} orbifold models have pairs of massless fields with conjugate representations R and $\overline{R}$ including massless fields with real representations. Such pairs of fields are very important for study on flat directions because VEVs of these fields can lead to D-flatness.

Here $\hat{U}_3$ is a subsector in the untwisted sector with the momentum which satisfies $\sum_t p^t v^t = 1/2$. The subsector $\hat{U}_3$ corresponds to the U_3 sector in Z_4, Z_6-II, Z_8-II and Z_{12}-II orbifolds, which have $v^3 = 1/2$ (mod Z). If P^I satisfies

$\sum_I P^I V^I = 1/2 \pmod{Z}$ for the Z_N invariant condition (16) in the $\hat{U}_3$ sector, $-P^I$ also satisfies $\sum_I P^I V^I = 1/2 \pmod{Z}$. When P^I corresponds to a particle with R representation, $-P^I$ corresponds to a particle with $\overline{R}$ representation. Thus pairs of R and $\overline{R}$ appear in massless spectra if one of them can appear.

Similar situations happen in the T_n sector of Z_{2n} orbifold models. Suppose that a shifted momentum $P^I + nV^I$ satisfies the massless condition (14). Then we always have the $E_8 \times E_8'$ momentum $P'^I = -P^I - 2nV^I$ which sits on the $E_8 \times E_8'$ lattice because of $2nV^I = 0 \pmod{\Gamma_{E_8 \times E_8'}}$, and $P'^I + nV^I$ satisfies the same massless condition (14). Thus conjugate pairs R and $\overline{R}$ satisfy the massless condition (14) at the same time.

Next we study GSO phases for these conjugate pairs with $P^I + nV^I$ and $P'^I + nV^I$. Since massless T_n sectors have $p^t + nv^t = 1/2(1,1,0,0)$ in Z_{2n} orbifold models, we have

$$\delta \equiv 2n \sum_t (p^t + nv^t) v^t = 1, 1, -1, -1, 2, -2 \text{ and } 3, \tag{24}$$

for Z_4, Z_6-I, Z_6-II, Z_8-I, Z_8-II, Z_{12}-I and Z_{12}-II orbifold models, respectively. Further the modular invariance requires

$$\sum_t (v^t)^2 - \sum_I (V^I)^2 = \frac{m}{n}, \tag{25}$$

where m is an integer. If $P^I + nV^I$ leads to $\tilde{\Delta}_n = (\omega_{(n)})^k$, i.e.

$$\sum_t (P^I + nV^I) V^I = \frac{\delta}{2n} - \frac{m}{2} + \frac{k}{n} \mod Z, \tag{26}$$

where $\omega_{(n)} \equiv \exp(2\pi i/n)$ and $k = 0, 1, ..., n-1$, then $P'^I + nV^I$ satisfies

$$\sum_t (P'^I + nV^I) V^I = -\frac{\delta}{2n} + \frac{m}{2} - \frac{k}{n} \mod Z. \tag{27}$$

Hence the charge conjugation $(P + nV) \to -(P + nV)$ transforms $\tilde{\Delta}_n$ as ‡

$$\tilde{\Delta}_n = \omega_{(n)}^k \longrightarrow \tilde{\Delta}_n = (\omega_{(n)})^{-k-\delta}. \tag{28}$$

Therefore pairs of R and $\overline{R}$ appear in massless spectra of the T_n sector if one of them can appear.§ The second column of Table 1 shows explicitly this transformation of $\tilde{\Delta}_n$ (28).

‡ This charge conjugation is similar to the CP transformation defined for orbifold models in Ref.[22]. In this CP transformation the 6-dimensional compact space and its supersymmetric space are transformed simultaneously through their parity reflections, so that $\tilde{\Delta}_n$ transforms into $1/\tilde{\Delta}_n$ under CP and Δ_n is invariant.

§ As an exception, conjugate spinor representations such as $SO(16)$ do not appear because the adjoint representation in E_8 does not include such conjugate spinor representations. However representations such as the $SO(16)$ spinor include both of $P + nV$ and $-(P + nV)$ for Eqs. (1) and (2), as we shall show later.

Note that if these R and $\overline{R}$ fields correspond to different values of $\tilde{\Delta}_n$, their degeneracies are, in general, different from each other. For example, we have 10 massless states with R and $\Delta = 1$, and 6 states with its conjugate representation $\overline{R}$ and $\Delta = -1$ in the T_2 sector of Z_4 orbifold models. The third column of Table 1 shows the common number of degeneracy factors for $\tilde{\Delta}_n = \omega^k_{(n)}$ and $\tilde{\Delta}_n = (\omega_{(n)})^{-k-\delta}$.

Moreover it is notable that massless fields with real representations such as 56 of E_7 can appear only if the value of $\tilde{\Delta}_n$ for $P^I + nV^I$ is same as one for $P'^I + nV^I$, i.e., $(\omega_{(n)})^k = (\omega_{(n)})^{-k-\delta}$. Because all fields in a multiplet should have the same value for $\tilde{\Delta}_n$ and a real representation includes both of $P^I + nV^I$ and $P'^I + nV^I$. For example, the T_2 sector of Z_4 orbifold models cannot have a real 56 representation of E_7 as a massless field. Such real representations can appear in T_3 of Z_6-I and II, T_4 of Z_8-II and T_6 of Z_{12}-I.

3.2 Classification of shift vectors

In the previous subsection, it is shown that T_n sectors of Z_{2n} orbifold models as well as the $\hat{U}_3$ sector can have conjugate pairs, R and $\overline{R}$. It is useful to classify shifts V^I in terms of $V^I_{(n)} \equiv nV^I$ in order to classify T_n sectors of Z_{2n} orbifold models. For any shift, $V^I_{(n)}$ satisfies $2V^I_{(n)} = 0$ (mod $\Gamma_{E_8 \times E_8}$). Thus $V^I_{(n)}$ should be a shift with the order 2. There are only three independent shifts with the order 2 for each E_8 lattice. These three shifts with the order 2, V^I_2, are classified in terms of $\sum_I (V^I_2)^2 = 0, 1/2$ or 1, where I runs from 1 to 8 or from 9 to 16. These shifts V^I_2 are written as

$$(0,0,0,0,0,0,0,0), \tag{29}$$

$$\tfrac{1}{2}(1,1,0,0,0,0,0,0), \tag{30}$$

$$(1,0,0,0,0,0,0,0), \tag{31}$$

up to E_8 rotations. The first shift does not break the E_8 group, while the others, Eqs. (30) and (31) break the E_8 group into $E_7 \times SU(2)$ and $SO(16)$, respectively.

Here we classify shifts V^I for E_8 in terms of corresponding $V^I_{(n)}$ into three classes, Classes 1, 2 and 3. Shifts V^I in Classes 1, 2 and 3 have $V^I_{(n)}$ which are equivalent to Eqs. (29), (30) and (31), respectively. All independent shifts of Z_N orbifold models for E_8 are obtained in Ref. [15]. Here we explicitly classify E_8 shifts for Z_4 and Z_6 orbifold models [23] into three classes.

First we discuss Z_4 orbifold models, whose independent E_8 shifts are shown in the second column of Table 2. Through the above classification, shifts V^I are classified by $2V^I$ as follows.

Class 1: $2V = (0, \cdots, 0)$ mod the E_8 lattice. This class includes the following three shifts;

$$\#0 \quad V = (0, \cdots, 0), \quad V^2 = 0, \tag{32}$$

$$\begin{aligned} &\#1 \quad V=(2,2,0,\cdots,0)/4, \quad V^2=1/2, && (33)\\ &\#4 \quad V=(1,0,\cdots,0), \quad V^2=1, && (34)\end{aligned}$$

where the number of the shift corresponds to that in Table 2. This class of the shifts leads to no massless matter field in U_1 and U_2 sectors ($\sum_I P^I V^I = 1/4$).

Class 2: $2V = (1,1,0,\cdots,0)/2$ or $2V=(1,-1,0,\cdots,0)/2$ mod the E_8 lattice.

$$\begin{aligned} &\#2 \quad V=(1,1,0,\cdots,0)/4, \quad V^2=1/8, && (35)\\ &\#3 \quad V=(2,1,1,0,\cdots,0)/4, \quad V^2=3/8, && (36)\\ &\#6 \quad V=(3,1,0,\cdots,0)/4, \quad V^2=5/8, && (37)\\ &\#8 \quad V=(3,1,\cdots,1,0,0)/4, \quad V^2=7/8. && (38)\end{aligned}$$

Note that $2V = (1,1,0,\cdots,0)/2$ and $2V = (1,-1,0,\cdots,0)/2$ are equivalent Z_2 division of E_8. This class of the shifts leads to 56 massless states satisfying $\sum_I P^I V^I = 1/4$ in each of U_1 and U_2.

Class 3: $2V = (1,0,\cdots,0)$ mod the E_8 lattice.

$$\begin{aligned} &\#5 \quad V=(2,0,\cdots,0)/4, \quad V^2=1/4, && (39)\\ &\#7 \quad V=(2,2,2,0,\cdots,0)/4, \quad V^2=3/4, && (40)\\ &\#9 \quad V=(1,\cdots,1,-1)/4, \quad V^2=1/2. && (41)\end{aligned}$$

This class of the shifts leads to 64 massless states satisfying $\sum_I P^I V^I = 1/4$ in each of U_1 and U_2.

We can show the reason why shifts in one class lead to the same number of the U_1 and U_2 massless fields ($\sum_I P^I V^I = 1/4$) as follows. For each shift we can write $2V^I$ as

$$2V^I = V_2^I + P_V^I, \tag{42}$$

where V_2^I is one of Eqs. (29), (30) and (31), and P_V^I sits on the E_8 lattice. Now we discuss the following equation:

$$\sum_I P^I (2V)^I = \frac{1}{2}, \quad (\text{mod Z}). \tag{43}$$

Because of $\sum_I P^I P_V^I =$ integer, the number of the E_8 roots P^I satisfying Eq. (43) depends only on $\sum_I P^I V_2^I$, but not on $\sum_I P^I P_V^I$. Thus all of the shifts in one class give the same number of P^I satisfying Eq. (43). These E_8 roots P^I satisfy $\sum_I P^I V^I = \pm 1/4$ (mod Z). It is obvious that if $\sum_I P^I V^I = 1/4$ (mod Z), its "conjugate" momentum $(-P^I)$ satisfies $\sum_I (-P^I) V^I = -1/4$ (mod Z). Hence the number of P^I satisfying $\sum_I P^I V^I = 1/4$ (mod Z) is a half of the number of P^I satisfying Eq. (43). Therefore all of the shifts in a class have the same number of massless U_i matter fields for $i = 1, 2$.

In the same way, E_8 shifts are classified into the following three classes for Z_6 orbifold models. The shifts, gauge groups and untwisted sectors are given in Table 3 explicitly.

Class 1: $3V = (0, \cdots, 0)$ mod the E_8 lattice. This class includes the following five shifts; #0, #3, #4, #9 and #17 in Table 3. This class of the shifts leads to no massless matter field in the sector with $\sum_I P^I V^I = 1/6$ or $3/6$. In this case, shifts are written as $V^I = P_V^I/3$, where P_V^I sits on the E_8 lattice. Thus the inner product $\sum_I P^I V^I$ should satisfy $\sum_I P^I V^I = m/3$, where m is an integer. That is the reason why this class of shifts have no matter fields with $\sum_I P^I V^I = 1/6$ or $3/6$.

Class 2: $3V = (1, 1, 0, ..., 0)/2$ mod the E_8 lattice. This class includes the following shifts; #1, #2, #5, #6, #11, #12, #14, #15, #20, #22, #23, and #24 in Table 3.

Class 3: $3V = (1, 0, ..., 0)$ mod the E_8 lattice. This class includes the following shifts; #7, #8, #10, #13, #16, #18, #19, #21, #25 and #26 in Table 3.

In the same way, E_8 shifts of Z_8 and Z_{12} orbifold models [24] ¶ can be classified into three classes. In general, shifts of Classes 2 and 3 break the E_8 into subgroups of $E_7 \times SU(2)$ and $SO(16)$, respectively.

3.3 Classification of T_n sector

In the previous subsection, shifts for E_8 are classified into three classes. Here we use this classification to study T_n sectors in Z_{2n} orbifold models. Combinations of E_8 and E_8' shifts are constrained due to the modular invariance Eq. (3). As discussed in the previous subsection, both of E_8 and E_8' shifts for Z_{2n} orbifold models, V^J ($J = 1 \sim 8$) and V^K ($K = 9 \sim 16$), are written as

$$nV^J = V_2^J + P_V^J, \quad nV^K = V_2^K + P_V^K, \tag{44}$$

where P_V^I sits on $\Gamma_{E_8 \times E_8'}$ and V_2^I is obtained as Eqs. (29), (30) and (31). They satisfy

$$\sum_{J=1}^{8} (V_2^J)^2 = \frac{m_{E_8}}{2}, \quad \sum_{K=9}^{16} (V_2^K)^2 = \frac{m_{E_8'}}{2}, \tag{45}$$

where $m_{E_8}, m_{E_8'} = 0, 1$ and 2 for Classes 1, 2 and 3, respectively. For T_n sectors of $Z_{N=2n}$ orbifold models, we obtain the following equation;

$$N \sum_{I=1}^{16} (V^I)^2 = \frac{m_{E_8} + m_{E_8'} + 2\ell}{n}, \tag{46}$$

where $\ell =$ integer, because $2V_2^I$ sits on the E_8 or E_8' lattice, i.e. $2\sum_I V_2^I P_V^I =$ integer. On the other hand, we use explicit values of v^i to find

$$nN \sum_i (v^i)^2 = \text{odd}, \tag{47}$$

¶See also Ref.[25].

for all $Z_{N=2n}$ orbifold models. Thus the modular invariance (3) requires $m_{E_8}+m_{E_8'}$ should be odd and allows the combinations where only one of the E_8 shift and the E_8' shift belongs to Class 2, i.e. combinations of Classes 2 & 1 and 3 & 2. The other four combinations are forbidden. Note that the Z_{2n} orbifold models with the unbroken $E_8 \times E_8'$ gauge group are not allowed, while the Z_3 orbifold model with the unbroken $E_8 \times E_8'$ gauge group are allowed.

Then we have only two types of T_n massless spectra of Z_{2n} orbifold models. In one type (Classes 2 & 1), the T_n massless condition for Z_{2n} orbifold models (14) is satisfied with 28 momenta $P + nV$ for $N_L^{(n)} = 0$ as

$$\begin{aligned} &(-1/2, -1/2, \pm 1, 0, \cdots, 0)(0, \cdots, 0), \\ &(1/2, 1/2, 0, \underline{\pm 1, 0, 0, 0, 0})(0, \cdots, 0), \\ &(0, 0, 1/2, \pm 1/2, \cdots, \pm 1/2)(0, \cdots, 0), \end{aligned} \tag{48}$$

and their "conjugates" $-(P + nV)$, and two momenta $(P + nV)$ for $N_L^{(n)} = 1/2$ as

$$(\underline{1/2, -1/2}, 0, \cdots, 0)(0, \cdots, 0), \tag{49}$$

in the base where $nV = 1/2(1, -1, 0, \cdots, 0)(0, \cdots, 0)$ mod the $E_8 \times E_8$ lattice. Totally the 56(=28+28) states correspond to a 56 representation of E_7 if it is unbroken. And two states with $N_L^{(n)} = 1/2$ correspond to a doublet representation of $SU(2)$. In general, this type has a gauge group $\tilde{G}(E_7) \times \tilde{G}(SU(2)) \times \tilde{G}(E_8')$, where $\tilde{G}(G)$ denotes a subgroup of G. Thus T_n sectors of Z_{2n} orbifold models do not have massless matter fields with any nontrivial representations under $\tilde{G}(E_8')$.

For example, the modular invariance allows twelve Z_4 orbifold models as shown in Table 4, where the third column shows combinations of E_8 shifts and E_8' shifts as the corresponding numbers of the first column in Table 2. Massless states of T_1 and T_2 are shown in the fourth and fifth columns of Table 4. Models of No. 1, 2, 3, 8, 9 and 10 in Table 4 correspond to combinations of shifts Classes 2 & 1. Actually all of these T_2 sectors have the following massless states;

$$N_L^{(2)} = 0, \quad \tilde{\Delta}_2 = 1, \quad 10 \times 28, \tag{50}$$

$$N_L^{(2)} = 0, \quad \tilde{\Delta}_2 = -1, \quad 6 \times 28, \tag{51}$$

$$N_L^{(2)} = 1/2, \quad \tilde{\Delta}_2 = 1, \quad 2 \times 10 \times 2, \tag{52}$$

$$N_L^{(2)} = 1/2, \quad \tilde{\Delta}_2 = -1, \quad 2 \times 6 \times 2, \tag{53}$$

where the degeneracy factor 10 (6) for $\tilde{\Delta}_2 = 1(-1)$ is generated by fixed points and the degeneracy factor 2 for $N_L^{(2)} = 1/2$ is generated by two oscillators corresponding to the first and second planes, $N_{L1}^{(2)}$ and $N_{L2}^{(2)}$. These 28 states correspond to (27 + singlet) in unbroken E_6 or 28 antisymmetric representation in unbroken $SU(8)$.

In the other type (Classes 3 & 2), the massless condition for T_n of Z_{2n} orbifold models (14) is satisfied with 16 momenta $P+nV$ for $N_L^{(n)}=0$ as

$$(\underline{\pm 1,0,\cdots,0})(1/2,-1/2,0,\cdots,0), \tag{54}$$

and their "conjugates" $-(P+nV)$ in the base where

$$nV=(1,0,\cdots,0)(1/2,-1/2,0,\cdots,0) \quad (\mathrm{mod}\ \Gamma_{E_8\times E_8'}). \tag{55}$$

These 32(=16+16) states correspond to $(16_v,2)$ under $SO(16)\times SU(2)'$ if it is not broken. There is no massless matter fields with nonvanishing $N_L^{(n)}$. In general, this type has a gauge group $\tilde{G}(SO(16))\times\tilde{G}(SU(2)')\times\tilde{G}(E_7')$, The above momenta do not have any quantum numbers under $\tilde{G}(E_7')$.

For example, models of No. 4, 5, 6, 7, 11 and 12 in Table 4 of Z_4 orbifold models correspond to this type. These T_2 sectors have the following massless spectrum;

$$N_L^{(2)}=0, \quad \tilde{\Delta}_2=1, \quad 10\times 16, \tag{56}$$
$$N_L^{(2)}=0, \quad \tilde{\Delta}_2=-1, \quad 6\times 16. \tag{57}$$

These 16 states correspond to 14 + 1 + 1 for unbroken $SO(14)$, (10,1) and (1,6) for unbroken $SO(10)\times SU(4)$ or (8,2) for unbroken $SU(8)\times SU(2)'$.

Taking into account the unbroken gauge groups, we can classify twelve Z_4 orbifold models of Table 4 as

$$1) \quad E_6\times SU(2)\times U(1)\times\tilde{G}(E_8') \text{ models}, \tag{58}$$
$$2) \quad SU(8)\times SU(2)\times\tilde{G}(E_8') \text{ models}, \tag{59}$$
$$3) \quad SO(14)\times U(1)\times U(1)'\times\tilde{G}(E_7') \text{ models}, \tag{60}$$
$$4) \quad SO(10)\times SU(4)\times U(1)'\times\tilde{G}(E_7') \text{ models}, \tag{61}$$
$$5) \quad SU(8)\times U(1)\times SU(2)'\times\tilde{G}(E_7') \text{ models}, \tag{62}$$

where the first and second types of models correspond to the combination Classes 2 & 1 and the other models correspond to the combination Classes 3 & 2. The latter type of models include anomalous $U(1)$ symmetries.[‖]

In the same way, we can classify T_n sectors of Z_{2n} orbifold models into two types, combinations of Classes 2 & 1 and 3 & 2. That is very important. There are many independent Z_N orbifold models, e.g., 58 Z_6-I, 61 Z_6-II, 246 Z_8-I, 248 Z_8-II, 3026 Z_{12}-I and 3013 Z_{12}-II orbifold models [4], but these are classified into only the above two types by the structure of T_n sectors. For example, T_3 sectors of all Z_6-I and II orbifold models are shown in Table 5 and 6, where $G_{6,3,2}$ denotes $SU(6)\times SU(3)\times SU(2)$. The third columns of Table 5 and 6 show combinations

[‖]This fact might provides a condition that an anomalous $U(1)$ symmetry appears. Some conditions for absence of anomalous $U(1)$ are shown in Ref.[26].

of E_8 shifts and E_8' shifts as the corresponding numbers of the first column in Table 3. The combinations of Classes 2 & 1 correspond to the following Z_6-I orbifold models;

$$\begin{aligned} &\text{No.}1, 2, 3, 4, 9, 10, 12, 13, 17, 19, 20, \\ &\quad 26, 27, 33, 39, 41, 44, 45 \text{ and } 49, \end{aligned} \tag{63}$$

and the following Z_6-II orbifold models;

$$\begin{aligned} &\text{No.}1, 2, 3, 4, 5, 6, 11, 12, 15, 20, 21, 25, \\ &\quad 26, 27, 34, 35, 41, 44, 46, 49 \text{ and } 50. \end{aligned} \tag{64}$$

The other Z_6 orbifold models correspond to Classes 3 & 2. Every massless state corresponds to Eq.(48), (49), (54) or their "conjugates". However, degeneracy factors are not so simple as those of T_2 of Z_4 orbifold models. Because the T_3 sectors of Z_6-I and II orbifold models have three values for $e^{i\gamma}$ (11), i.e. 1, ω and ω^2, where ω is the third root of unity, and these values of $e^{i\gamma}$ in Z_6-I (II) orbifold models correspond to degeneracy factors, 6, 5 and 5 (8, 4 and 4), respectively. Further, in general, each multiplet can have a different value of $\tilde{\Delta}_{(n)}$ from others. For example, in the Z_6-I orbifold model with No. 52, $(6, 1, 1; 1, 2)$ and its "conjugate" $(\overline{6}, 1, 1; 1, 2)$ have $\tilde{\Delta}_3 = 1$ and ω^2, leading to degeneracy factors 6 and 5, respectively. This relation of $\tilde{\Delta}_3$ is consistent with Eq.(28). On the other hand, $(1, 2, 2; 1, 2)$ has $\tilde{\Delta}_3 = \omega$ and degeneracy factor 5. However, in any case we can find at least five pairs of $P+3V$ and $-(P+3V)$ as shown in Table 1. Similarly we can find at least four conjugate pairs in the T_3 sector of Z_6-II orbifold models making use of the relation (28) and corresponding degeneracy factors. Further T_4 of Z_8-I and II orbifold models and T_6 of Z_{12}-I and II orbifold models have at least three and two conjugate pairs, respectively. Moreover we find at least four (three) conjugate pairs in T_4 (T_6) of Z_8-II (Z_{12}-I) orbifold models if corresponding $P + nV$ or $-(P + nV)$ leads to $\tilde{\Delta}_n = 1$ (ω^m). These common numbers of degeneracy factors for conjugate pairs are shown in the third column of Table 1. These numbers are very important for the study of gauge symmetry breaking by flat directions.

4 Flat directions

4.1 Generic flat directions in T_n and $\hat{U}_3$ sectors of Z_{2n} orbifold models

Some conjugate pairs of T_n matter fields with momenta $P+nV$ and $-(P+nV)$, $T_{n,P+nV}$ and $T_{n,-(P+nV)}$, can appear in Z_{2n} orbifold models including conjugate pairs of the $\hat{U}_3$ sector . VEVs of these fields are important for the study on flat

directions, because they lead to the following D-flat direction;

$$\langle T_{n,P+nV}\rangle = \langle T_{n,-(P+nV)}\rangle \neq 0. \tag{65}$$

If this direction (65) is also a flat direction for F-terms, a rank of a gauge group is reduced by a degeneracy of these conjugate pairs. In this subsection, we study whether this direction (65) is really a flat direction for the superpotential W derived from orbifold models. Flat directions mean

$$\langle W\rangle = \langle \frac{\partial W}{\partial \chi}\rangle = 0, \tag{66}$$

as well as vanishing D-terms where χ is any chiral superfield.

Let us consider flat directions (65) in T_n sectors of Z_{2n} orbifold models. In this case, we have to examine $(T_n)^\ell$ and $\chi(T_n)^\ell$ couplings. Here we restrict ourselves to renormalizable couplings in W for a while. In this case, the point group selection rule allows only $(T_n)^2$ couplings and $U_i(T_n)^2$ couplings among couplings relevant to flat directions (65) of T_n. The T_n sectors have the $SO(6)$ momentum $p+nv=(1,1,0)/2$ and the total momentum of $U_i(T_n)^2$ couplings is reduced by $(1,1,1)$ through twice supertransformation to obtain the $V_FV_FV_B$ or $V_AV_BV_B$ form. Thus only the $U_3(T_n)^2$ coupling is allowed because of the $SO(6)$ momentum conservation. Further a product of $e^{i\gamma}$ for coupled states should be unity. That means a product of corresponding $\tilde{\Delta}_n$ should be unity, if their coupling is allowed. Therefore the conjugate pairs, R and $\overline{R}$, cannot couple, because a product of the corresponding values for $\tilde{\Delta}_n$ is obtained as $\omega^k_{(n)}\omega^{-k-\delta}_{(n)}=\omega^{-\delta}_{(n)}$ due to the relation (28), and $\delta/n \neq$ integer (24). Moreover the space group selection rule requires that coupled T_n states sit the same fixed points because of Eq.(22).

For example, the Z_4 orbifold models have the following superpotential;

$$\begin{aligned} W &= \sum_a U_{3,P_U}T_{n,a,P_1+nV}T_{n,a,P_2+nV} \\ &+ \sum_b U_{3,P_U}T_{n,b,-(P_1+nV)}T_{n,b,-(P_2+nV)}, \end{aligned} \tag{67}$$

where a (b) denotes 10 (6) states with $e^{i\gamma}=1$ (-1) and coupling strengths are omitted. Note that in W the same 6-dimensional states are allowed to couple. Thus Eq.(65) is a flat direction if there does not exist the U_3 field satisfying $P_U \pm (P_1+nV) \pm (P_2+nV)=0$ (i.e. gauge invariance) in massless spectra. Here we make six pairs of conjugate states with $\tilde{\Delta}_2=1$ and -1 and then consider the following VEVs;

$$\langle T_{n,a=c,P_c+nV}\rangle = \langle T_{n,b=c,-(P_c+nV)}\rangle \neq 0, \tag{68}$$

where $c=1\sim 6$ and $T_{n,a=c,P_c+nV}$ are six of ten degenerate states with $e^{i\gamma}=1$. Eq.(68) implies that for each "fixed point" c only one field (in a multiplet) with

P_c+nV develops its VEV. In this case, couplings relevant to flat directions are the couplings (67) with $P_1+nV=P_2+nV$. This momentum is one of Eqs.(48), (54) and their "conjugates". Among Eqs. (48), (54) and their "conjugates", there is no momentum for T_n satisfying $P_U+2(P+nV)=0$ with the E_8 root vector P_U, i.e. (1) and (2). Thus there is no U_3 massless matter field which have a coupling with $(T_{n,a=c,P+nV})^2$ or $(T_{n,b=c,-(P+nV)})^2$, due to the E_8 momentum conservation. That implies that the group $\tilde{G}(E_7)$ ($\tilde{G}(SO(16))$ could break along this flat direction (68) reducing its rank by at least 6 in the type of models with Classes 2 & 1 (3 & 2) shifts. Also $\tilde{G}(SU(2)')$ in the Classes 3 & 2 type of models could break. Further $\tilde{G}(SU(2))$ in the Classes 2 & 1 type of models could break through VEVs of the massless matter fields with the momenta (49) and $N_L^{(2)}=1/2$.

Similarly we can find flat directions (65) for the other Z_{2n} orbifold models. For example, the U_3 and T_3 sectors of Z_6-I orbifold models have the following superpotential;

$$\begin{aligned} W &= \sum_a U_{3,P_U} T_{n,\tilde{\Delta}=1,a,P_1+nV} T_{n,\tilde{\Delta}=1,a,P_2+nV} \\ &+ \sum_b U_{3,P_U} T_{n,\tilde{\Delta}=\omega,b,-(P_1+nV)} T_{n,\tilde{\Delta}=\omega^2,b,-(P_2+nV)}, \end{aligned} \tag{69}$$

where a (b) denotes 6 (5) states with $e^{i\gamma}=1$ (ω and ω^2). This potential also has the following flat direction;

$$\begin{aligned} \langle T_{3,\tilde{\Delta}=1,a=c,P_c+nV}\rangle &= \langle T_{3,\tilde{\Delta}=\omega,b=c,-(P_c+nV)}\rangle \neq 0, \\ \langle T_{3,\tilde{\Delta}=\omega^2,b,P_b+nV}\rangle &= \langle T_{3,\tilde{\Delta}=\omega^2,b,-(P_b+nV)}\rangle \neq 0, \end{aligned} \tag{70}$$

where $c=1\sim 5$. Along this direction a rank of a gauge group could break by at least 5. In a similar way, other T_n sectors of Z_{2n} orbifold models can have flat directions. Along these directions, ranks of gauge groups could reduce by the number of common degeneracy factors for conjugate pairs.

In addition to the flat directions for T_n, the $\hat{U}_3$ sector can have another flat direction as

$$\langle \hat{U}_{3,P}\rangle = \langle \hat{U}_{3,-P}\rangle \neq 0. \tag{71}$$

Because W does not include $\chi(\hat{U}_3)^\ell$ couplings, even if we take into account nonrenormalizable couplings. Thus this is a flat direction for all orders of W. Therefore if a gauge group has these pairs of massless matter fields in the $\hat{U}_3$ sector, this group breaks reducing its rank by the number of pairs. However both VEVs of T_n and $\hat{U}_3$ (68) and (71) are not always flat directions at the same time. Because that could lead to $\langle \partial W/\partial T_n\rangle \neq 0$ for $W=\hat{U}_3(T_n)^2$. Such a situation is model-dependent and we have to examine explicit $\hat{U}_3$ massless spectra of models.

Next we study effects of nonrenormalizable couplings on flat directions for T_n. We discuss $\chi(T_n)^\ell$ couplings. Among this type of couplings, the selection

rule due to the point group allows $U_i(T_n)^{2\ell}$ couplings. For example, $U_3(T_n)^{2\ell}$ couplings has the total $SO(6)$ momentum $(\ell-1,\ell-1,0)$ in $V_{-1/2}V_{-1/2}(V_{-1})^{2\ell-1}$ form. We use the picture changing operator (2ℓ) times to change this form into $V_{-1/2}V_{-1/2}V_{-1}V_0^{2\ell-2}$ form, where the total $SO(6)$ momentum should be conserved for allowed couplings. Thus the obtained couplings include the oscillators as

$$(\partial X_{1(n)})^{\ell-1}(\partial X_{2(n)})^{\ell-1}. \tag{72}$$

This should be invariant under the Z_N twist of oscillators (23). That requires $\ell-1=$even. Thus the allowed couplings are obtained as $U_3(T_n)^{4m+2}$ couplings with $m=$ integer. The other $U_i(T_n)^\ell$ couplings $(i=1,2)$ are forbidden. Even if we take into account these $U_3(T_n)^{4m+2}$ couplings, we find the flat direction where only one pair develop their VEVs. Because in this case $U_3(T_{n,P+nV})^k(T_{n,-(P+nV)})^\ell$ couplings with $k+\ell=4m+2$ are dangerous couplings to lift the flatness, but we cannot obtain the nonzero E_8 root vector P_U satisfying $P_U+k(P+nV)-\ell(P+nV)$ for the momentum $P+nV$ (48), (54) or their "conjugates".

Similarly we investigate flat direction in the case with more than one pairs developing VEVs. For example, we discuss $U_3(T_2)^6$ couplings in Z_4 orbifold models. Suppose that in Eq.(68) VEVs are developed by two pairs of states with

$$P_1+2V=(1,0,\cdots,0)(1/2,-1/2,0,\cdots,0), \tag{73}$$

and $-(P_1+2V)$ for $c=1$,

$$P_2+2V=(0,1,0,\cdots,0)(1/2,-1/2,0,\cdots,0), \tag{74}$$

and $-(P_2+2V)$ for $c=2$, We assign these (73) and (74) to different 6-dimensional ground states. For simplicity, here we assign a conjugate pair of (P_c+2V) and $-(P_c+2V)$ to the 6-dimensional ground states with the same fixed points and the different values of $e^{i\gamma}$, e.g. $|e_2+ke_3\rangle \pm |e_1+e_2+ke_3\rangle$. Then we investigate $U_3(T_2)^6$ couplings. If these couplings include even fields of one conjugate pair, we can find no U_3 field to couple those fields. Because $2m(P_c+2V)$ of Eqs. (48), (54) or their "conjugates" is too large compared with the E_8 root vectors (1) and (2), which the U_3 states have, and by adding the other momenta, $-(P_1+2V)$, (P_2+2V) and $-(P_2+2V)$, one cannot reduce it into the E_8 roots except vanishing momentum. Note that the U_3 sector does not have vanishing momentum. Thus nonvanishing $U_3(T_2)^6$ couplings should include odd fields of one conjugate pair. For example, $(T_{2,c=1,P_1+2V})^2T_{2,c=1,-(P_1+2V)}T_{2,c=2,P_2+2V}(T_{2,c=2,-(P_1+2V)})^2$ couplings have the $E_8\times E_8'$ momentum $(1,-1,0,\cdots,0)(0.\cdots,0)$. This momentum is included in the E_8 root vectors and the U_3 sector could have it in a certain case, although it is model-dependent whether a model really has this momentum. We have to discuss the space group selection rules for this coupling. Here corresponding fixed points are denoted as (12)

$$\begin{aligned}&(\theta^2, i_1e_1+j_1e_2+k_1e_3+\ell_1e_4) \quad \text{for} \quad \pm(P_1+2V), \\ &(\theta^2, i_2e_1+j_2e_2+k_2e_3+\ell_2e_4) \quad \text{for} \quad \pm(P_2+2V).\end{aligned} \tag{75}$$

Due to the space group selection rules (22), these couplings are allowed if they satisfy

$$3(i_1 + i_2) = \text{even}, \quad 3(j_1 + j_2) = \text{even}, \tag{76}$$
$$3(k_1 + k_2) = \text{even}, \quad 3(\ell_1 + \ell_2) = \text{even}. \tag{77}$$

That implies $i_1 = i_2$, $j_1 = j_2$, $k_1 = k_2$ and $\ell_1 = \ell_2$, i.e., the exactly same fixed point. Thus the different fixed points $c = 1, 2$ have no $U_3(T_2)^6$ coupling. Similarly we can show that, in general, T_n sectors of Z_{2n} orbifold models have no $U_3(T_n)^{4m+2}$ couplings which include two conjugate pairs with any momenta of (48), (54) and their "conjugates". Because for fixed points like Eq.(75) this type of couplings requires

$$(\text{odd}) \times i_1 + (\text{odd}) \times i_2 = \text{even}, \tag{78}$$

and similar equations for j, k, and ℓ, i.e., the same fixed point. Thus for all orders the direction of Eqs. (65) and (68) is a flat direction, which reduces a rank of a gauge group by two.

We extend the above analysis to the case where three conjugate pairs develop their VEVs as Eq.(68) ($c = 1, 2, 3$) in Z_4 orbifold models. In addition to the fixed points (75), we denote the third fixed point as

$$(\theta^2, i_3e_1 + j_3e_2 + k_3e_3 + \ell_3e_4), \tag{79}$$

and its momenta as $\pm(P_3 + 2V)$. In a similar way to the above discussion, $U_3(T_2)^{4m+2}$ couplings should include odd fields of each conjugate pair in T_2, due to the $E_8 \times E_8'$ momentum conservation. Then the total number of the fields is odd. That conflicts with $4m + 2$. Therefore such couplings are not allowed. This can be extended into any case where odd conjugate pairs develop their VEVs in T_n sectors of Z_{2n} orbifold models. In this case, $U_3(T_n)^{4m+2}$ couplings are not allowed.

Further we consider the case where four conjugate pairs develop their VEVs. In addition to the above, we denote the fourth fixed point as

$$(\theta^2, i_4e_1 + j_4e_2 + k_4e_3 + \ell_4e_4), \tag{80}$$

and its momentum $\pm(P_4 + nV)$. In this case, the space group selection rule as well as the $E_8 \times E_8'$ momentum conservation requires

$$\sum_{d=1}^{4} (\text{odd}) \times i_d = \text{even}, \tag{81}$$

and similar equations for j_d, k_d and ℓ_d. We can find combinations of fixed points not to satisfy these equations, e.g.,

$$(i, j, k, \ell) = (1, 0, 1, 1),\ (1, 0, 1, 0),\ (0, 0, 0, 0) \text{ and } (0, 0, 1, 1). \tag{82}$$

Such a combination leads to a flat direction, which break a gauge group reducing its rank by four.

Moreover we can find some combinations of fixed points not to satisfy the following equation;

$$\sum_{d=1}^{6}(\text{odd}) \times i_d = \text{even}, \tag{83}$$

where i_5 and i_6 correspond to the fifth and sixth fixed points, or similar equations for j_d, k_d and ℓ_d. Therefore the direction (68) with $c = 1 \sim 6$ is still a flat direction for all orders of nonrenormalizable couplings $U_3(T_2)^{4m+2}$. As results, gauge groups $\tilde{G}(E_7) \times \tilde{G}(SU(2)) \times \tilde{G}(E_8')$ and $\tilde{G}(SO(16)) \times \tilde{G}(SU(2)') \times \tilde{G}(E_7')$ in Z_4 orbifold models can break through the flat directions at least into $SU(2)_{E_7} \times \tilde{G}(E_8')$ and $SU(3)_{SO(16)} \times \tilde{G}(E_7')$, where $SU(2)_{E_7}$ and $SU(3)_{SO(16)}$ denotes $SU(2)$ and $SU(3)$ subgroups in the original groups E_7 and $SO(16)$, respectively. Similarly these gauge groups in other orbifold models break reducing their ranks by the numbers shown in the third column of Table 1 as the common number of degeneracy factors for conjugate pairs of $\tilde{\Delta}_n = \omega_{(n)}^k$ and $\tilde{\Delta}_n = (\omega_{(n)})^{-k-\delta}$. Thus drastic symmetry breaking could happen.

These results have important phenomenological implications. For example, we cannot expect appearance of the standard model gauge group $SU(3) \times SU(2) \times U(1)$ in the E_7 sector of the Z_4 orbifold models with $\tilde{G}(E_7) \times \tilde{G}(SU(2)) \times \tilde{G}(E_8')$. In this case, we have to concentrate the E_8' sector for candidates of the standard model gauge group. On the other hand, ranks of gauge groups in Z_8 and Z_{12} orbifold models are reduced not so much along the generic flat directions we have discussed because the degeneracy factors are small compared with those of Z_4 and Z_6 orbifold models as given in Table 1. For example, in these orbifold models it might be possible to derive the standard model gauge group in the E_7 sector from the $\tilde{G}(E_7) \times \tilde{G}(SU(2)) \times \tilde{G}(E_8')$ model, although explicit models could have more accidental flat directions leading to further symmetry breaking.

4.2 Example

We investigate the previous results on the flat directions using an explicit model. Here we take the $E_6 \times SU(2) \times U(1) \times \tilde{G}(E_8')$ models (58) of Z_4 orbifold models. As shown in Tables 2 and 4, this type of models have the following T_2 massless matter fields;

$$10 \times [(\overline{27}, 1)_1 + (1,1)_{-3}] \quad (\tilde{\Delta}_2 = 1, N_L^{(2)} = 0), \tag{84}$$

$$6 \times [(27, 1)_{-1} + (1,1)_3] \quad (\tilde{\Delta}_2 = -1, N_L^{(2)} = 0), \tag{85}$$

$$20 \times (1,2)_0 \quad (\tilde{\Delta}_2 = 1, N_L^{(2)} = 1/2), \tag{86}$$

$$12 \times (1,2)_0 \quad (\tilde{\Delta}_2 = -1, N_L^{(2)} = 1/2), \tag{87}$$

and the following U_i massless matter fields;

$$U_1 : (\overline{27}, 2)_1 + (1, 2)_{-3}, \quad U_2 : (\overline{27}, 2)_1 + (1, 2)_{-3}, \tag{88}$$

$$U_3 : (\overline{27}, 1)_{-2} + (27, 1)_2, \tag{89}$$

under $E_6 \times SU(2) \times U(1)$. Further U_i's include some $\tilde{G}(E_8')$ matter fields without quantum numbers of $E_6 \times SU(2) \times U(1)$.

The superpotential in T_2 and U_3 sectors is written as

$$\begin{aligned} W_{T_2T_2U_3} &= \sum_a d_{ijk}(\overline{27}_{-2})^i_{U_3}(\overline{27}_1)^j_{T_2,a}(\overline{27}_1)^k_{T_2,a} + \sum_b d_{ijk}(27_2)^i_{U_3}(27_{-1})^j_{T_2,b}(27_{-1})^k_{T_2,b} \\ &+ \sum_a (27_2)^i_{U_3}(\overline{27}_1)^i_{T_2,a}(1_{-3})_{T_2,a} + \sum_b (\overline{27}_{-2})^i_{U_3}(27_{-1})^i_{T_2,b}(1_3)_{T_2,b}, \end{aligned} \tag{90}$$

where indices for $SU(2)$ are omitted, $(27)^i$ denotes the i-th element of a 27 multiplet and a and b denote the T_2 states with $\tilde{\Delta}_2 = 1$ and -1, respectively. Here d_{ijk} denotes the third rank antisymmetric invariant. This superpotential does not include the doublet fields. Hence the doublet fields always develop their VEVs in the flat direction.

Let us study the following D-flat direction;

$$< (\overline{27}_{-2})^i_{U_3} >=< (27_2)^i_{U_3} >= v_0^i, \tag{91}$$

$$< (\overline{27}_1)^i_{T_2,c(\Delta=1)} >=< (27_{-1})^i_{T_2,c(\Delta=-1)} >= v_c^i, \tag{92}$$

$$< (1_{-3})_{T_2,c(\Delta=1)} >=< (1_3)_{T_2,c(\Delta=-1)} >= u_c, \tag{93}$$

where $c = 1 \sim 6$. We have the following conditions for the F-flatness;

$$< \frac{\partial W}{\partial (1)_{T_2,c}} >= v_0^i v_c^i = 0, \tag{94}$$

$$< \frac{\partial W}{\partial (27)^j_{T_2,c}} >= d_{ijk} v_0^i v_c^k + v_0^j u_c = 0, \tag{95}$$

$$< \frac{\partial W}{\partial (27)^i_{U_3}} >= \textstyle\sum_c [d_{ijk} v_c^j v_c^k + v_c^i u_c] = 0. \tag{96}$$

Because of $d_{ijj} = 0$, one of the flat directions is obtained as

$$v_0^i = 0, \quad u_c = \delta_c^1, \quad v_c^i = \delta_c^i, \tag{97}$$

where $c \neq 1$ for the last equation. Along this direction, the gauge group $E_6 \times U(1)$ can break into $SU(2)_{E6}$, reducing its rank by six. That is consistent with the previous result. On the top of that there is the flat direction where a combination of three $\overline{27}_{T_2,a}$ ($a \neq c$ e.g. $a = 7, 8, 9$) develops their VEVs at the same time. In this case, $SU(2)_{E6}$ is also broken. After these breaking, some of T_1 matter fields gain masses through the $T_1T_1T_2$ couplings and the observable U_3 matter fields gain masses. However the $U_{1,2}$ matter fields remain massless. There is another flat direction, where $v_0^i \neq 0$. In this case, all of the observable $U_{1,2}$ fields gain masses through the $U_1U_2U_3$ couplings.

We investigate more explicitly which fields become massive through the above flat directions, using the Z_4 orbifold model of No.1 in Table 4 with the gauge group $E_6 \times SU(2) \times U(1) \times E_8'$. As shown in Table 4, this model has the following T_1 massless fields;

$$16 \times (\overline{27}, 1)_{-1/2} \quad (N_L^{(1)} = 0), \tag{98}$$

$$32 \times (1, 2)_{-3/2} \quad (N_L^{(1)} = 1/4), \tag{99}$$

$$80 \times (1, 1)_{3/2} \quad (N_L^{(1)} = 1/2). \tag{100}$$

(1) $U(1)$ breaking

This happens when a singlet pair in T_2 develops VEVs, i.e. $u_c \neq 0$. The singlet fields in T_1 gain masses through this breaking. Among the 80 singlets, the 16 singlets remain massless and these corresponding $N_{Li}^{(1)} = (1/4, 1/4, 0)$. Further through $W_{T_2T_2U_3}$ the two pairs of $(27 + \overline{27})$ of U_3 and T_2 obtain mass terms.

(2)$E_6 \rightarrow SO(10)$ breaking

Case I: This happens when a $(27 + \overline{27})$ pair in T_2 develop VEVs, i.e. $v_c^1 \neq 0$. We assign the first element $(27)^1$ as the $SO(10)$ singlet 1_4. Then 1_4 in 27 and 1_{-4} in $\overline{27}$ develop VEVs. Because of $d_{i11} = 0$, this satisfies the flatness conditions. Through this breaking, every 27 and $\overline{27}$ are decomposed into

$$27 = 1_4 + 10_{-2} + 16_1, \quad \overline{27} = 1_{-4} + 10_2 + \overline{16}_{-1}, \tag{101}$$

under $SO(10) \times U(1)_{E6}$. Through this breaking, every generation of the 10-fields in T_1 has a mass by $(T_1)^2 T_2$ couplings. There appear fourteen massless singlets without any quantum numbers for unbroken gauge groups in T_2. Note that for the whole gauge group nonvanishing v_c^1 leads to $E_6 \times U(1) \rightarrow SO(10) \times U(1)_I$, where the current of $U(1)_I$ is a linear combination of those for $U(1)_{E6}$ and the original $U(1)$, i.e., $J_{U(1)_I} = J_{U(1)E6} + 4J_{U(1)}$.

One can break $U(1)_I$ with nonvanishing u_c as (1). In this case the flatness conditions require $\sum_c v_c^1 u_c = 0$. That implies the Higgs fields breaking E_6 should sit on the different fixed point from one for the Higgs fields breaking $U(1)$. At this stage we have the following massless U_i spectrum under the unbroken $SO(10) \times SU(2)$;

$$U_1 : (\overline{27}, 2) + (1, 2), \quad U_2 : (\overline{27}, 2) + (1, 2), \tag{102}$$

the following massless T_1 spectrum;

$$16 \times [(1, 1) + (\overline{16}, 1)](N_L^{(1)} = 0), \tag{103}$$

$$32 \times (1, 2)(N_L^{(1)} = 1/4), \quad 16 \times (1, 1)(N_L^{(1)} = 1/2), \tag{104}$$

and the following massless T_2 spectrum;

$$8 \times (\overline{27}, 1)(\Delta = 1, N_L^{(2)} = 0), \quad 4 \times (27, 1)(\Delta = -1, N_L^{(2)} = 0), \tag{105}$$

$$9 \times (1, 1)(\Delta = 1, N_L^{(2)} = 0), \quad 5 \times (1, 1)(\Delta = -1, N_L^{(2)} = 0), \tag{106}$$

$$20 \times (1, 2)(\Delta = 1, , N_L^{(2)} = 1/2), \quad 12 \times (1, 2)(\Delta = -1, N_L^{(2)} = 1/2), \tag{107}$$

where 27 is a short notation for $1 + 10 + 16$.

Case II:This happens when a $(27+\overline{27})$ pair in U_3 develops VEVs, i.e. $v_0^1 \neq 0$. In a similar way to Case I, we can calculate massive modes. In this case, every 10-field in U_i and T_2 becomes massive. In addition, $(1,2)_{-3}$ and the $SO(10)$ singlets in $\overline{27}$ of $U_{1,2}$ become massive. The nonvanishing v_0^1 leads to the breaking $E_6 \times U(1) \to SO(10) \times U(1)_J$, where the current $J_{U(1)_J}$ is obtained as $J_{U(1)_J} = J_{U(1)E6} - 2J_{U(1)}$. Let us study the $U(1)_J$ breaking by nonvanishing u_c. In this case, however, the flatness conditions can not be satisfied with nonvanishing v_0^j and u_c at the same time. Thus in Case II there is no $U(1)_J$ breaking.

(3) $SO(10) \to SU(5)$ breaking

For the $SO(10)$ model without $U(1)_I$ obtained in Case I, we consider further breaking, where a pair of $(16 + \overline{16})$ develops VEVs, i.e. $v_{c=1}^1 \neq 0$, $v_{c=2}^2 \neq 0$ and $u_{c=3} \neq 0$ at the same time, where v_2^2 corresponds to a $SU(5)$ singlet in 16. Because of $d_{i12} = d_{i22} = 0$, these VEVs satisfy the flatness conditions.

Through this breaking, the gauge group breaks as $SO(10) \to SU(5)$. The representations 16 and $\overline{16}$ are decomposed as

$$16 = 1_{-5} + \overline{5}_3 + 10_{-1}, \quad \overline{16} = 1_5 + 5_{-3} + \overline{10}_1, \tag{108}$$

under $SU(5) \times U(1)_{SO(10)}$. Further 10 representation is decomposed as

$$10 = 5_2 + \overline{5}_{-2}. \tag{109}$$

Through this breaking, no matter fields obtain masses except the Higgs fields. For example, the $\overline{16}$-fields in T_1 cannot couple through $T_1T_1T_2$-coupling with the 16 or $\overline{16}$- field in T_2, to whose multiplet the Higgs field belongs. Thus the massless spectrum is changed not so drastically. In the above notation, 27 means under this gauge group as

$$27 = 1 + 1 + 5 + \overline{5} + \overline{5} + 10. \tag{110}$$

We can continue this symmetry breaking as

$$SU(5) \to SU(4) \to SU(3) \to SU(2),$$

using a 5 representation of subgroup $SU(5)$ in 16-multiplet of $SO(10)$, 4 and 3 representations of subgroups $SU(4)$ and $SU(3)$, respectively. Also this $SU(2)$ can be broken by one pair of doublets. Through this series of breakings, the number of T_1 massless field is not changed, since $16(\overline{16})^2$ coupling is forbidden. For T_2, the number of 27 representation is reduced by one leaving singlets under unbroken groups, as the rank of the gauge group is reduced by one.

Alternatively we can reduce the number of the T_1 massless fields in different types of breakings. For example, we can break $SU(5)$ by $10(= 5 + \overline{5}$ in $SU(5)$ base) of the $SO(10)$ multiplet in T_2. This allows the $10(\overline{16})^2$ coupling. In this breaking, we have the gauge group $SU(3) \times SU(2)$ and some of T_1 fields gain masses. We can investigate flat directions of other models explicitly.

4.3 Wilson lines

In addition to V^I, Wilson lines $a^I_{e^i}$ can be embedded into $\Gamma_{E_8\times E_8'}$ as [5, 27, 12]

$$(\theta^k, e^i) \to (kV^I, a^I_{e^i}). \tag{111}$$

The lattice vectors related through θ should correspond to an equivalent Wilson line, i.e. $a^I_{e^i} = a^I_{(\theta e)^i}$. For example, the first equation of (4) leads to the constraint that $2a^I_{e_a}$ $(a = 2, 4)$ should be on $\Gamma_{E_8\times E_8'}$ in Z_4 orbifold models. Further the second equation requires $a^I_{e_a} = 0$ (mod $\Gamma_{E_8\times E_8'}$) $(a = 1, 3)$. Similarly Eq. (5) leads to the conditions $2a^I_{e_5} = 2a^I_{e_6} = 0$ (mod $\Gamma_{E_8\times E_8'}$). It is remarkable that the states in the same linear combination to construct a θ-eigenstate like Eq. (11) have the same Wilson lines. The structure of Wilson lines depends on the Lie lattice to construct the orbifold, because θ transforms lattice vectors in a different way. Appendix A shows the Lie lattice leading to Wilson lines with the most degrees of freedom among the same Z_N orbifold models [27, 12].

For the models with Wilson lines a^I, the massless U fields should satisfy the following equation;

$$\sum_I P^I a^I = \text{integer}. \tag{112}$$

These Wilson lines could lead to smaller gauge groups and reduce the number of massless U matter fields. Hence Wilson lines are important to derive realistic models. If P^I satisfies Eq.(112), its conjugate $-P^I$ also satisfies it. Thus the $\hat{U}_3$ sector can have conjugate pairs, R and $\overline{R}$.

The T_k sector corresponding to Eq.(111) has the shifted momentum $(P+kV+a_{e^i})$. Its left-moving massless condition is written as

$$\frac{1}{2}\sum_{I=1}^{16}(P^I + kV^I + a^I_{e^i})^2 + N_L^{(k)} + c_k - 1 = 0. \tag{113}$$

Thus Wilson lines resolve degeneracy of T_k massless matter fields. The GSO phase with Wilson lines is obtained by replacing kV^I in Eq.(18) into $kV^I + a^I_{e^i}$. Further the constraint due to the modular invariance is written as

$$N\sum_{i=1}^{3}(kv^i)^2 - N\sum_{I=1}^{16}(kV^I + a^I_{e^i})^2 = \text{even}, \tag{114}$$

for each value of k.

Now let us consider conjugate pairs R and $\overline{R}$ in the T_2 sector of Z_4 orbifold models. The fixed points of Eq.(12) have the following $E_8 \times E_8'$ momenta;

$$P^I + 2V^I + ja^I_{e_2} + \ell a^I_{e_4}. \tag{115}$$

Since both of $a^I_{e_2}$ and $a^I_{e_4}$ are Wilson lines with the order 2, we have

$$\tilde{V}^I_{j,\ell} \equiv 2V^I + ja^I_{e_2} + \ell a^I_{e_4}, \quad 2\tilde{V}^I_{j,\ell} = 0 \ (\text{mod } \Gamma_{E_8\times E_8'}). \tag{116}$$

That implies if $P^I+\tilde{V}^I_{j,\ell}$ satisfies the massless condition (113), we always have $P' = -P^I-\tilde{V}^I_{j,\ell}$ which sits on $\Gamma_{E_8\times E_8'}$, and $P'^I+\tilde{V}^I_{j,\ell}$ satisfies the same massless condition for each combination of (j,ℓ). The transformation of $\tilde{\Delta}_n$ under charge conjugation is obtained in a way similar to the case without Wilson lines. Thus the T_2 sector can have conjugate pairs R and $\overline{R}$ for the states with same values of (j,ℓ). It is notable that these conjugate pairs appear in the states of linear combinations with the same content and different eigenvalues, e.g., $|e_2+ke_3\rangle + |e_1+e_2+ke_3\rangle$ and $|e_2+ke_3\rangle - |e_1+e_2+ke_3\rangle$, while in the case without Wilson lines we can make pairs of any combinations with $\tilde{\Delta}_2 = 1$ and -1 due to the degeneracy. Further generally conjugate pairs do not appear for the states with $\tilde{\Delta}_2 = 1$ which have no associated states with $\tilde{\Delta}_2 = -1$, e.g. $|ie_1+ke_3\rangle$.

Since $\tilde{V}^I_{j,\ell}$ is a shift with the order 2, each of its E_8 part ($I = 1 \sim 8$) and its E_8' part ($I = 9 \sim 16$) is classified three classes, Classes 1, 2 and 3 in the same way as the case without Wilson lines. Further the modular invariance (114) allows only two combinations of these Classes for $\tilde{V}^I_{j,\ell}$, Classes 2 & 1 and Classes 3 & 2. Thus the massless T_2 fields have the shifted $E_8\times E_8'$ momenta of Eqs. (48), (49), (54) and their "conjugates" for each (j,ℓ). Note that in one model the twisted matter fields of Classes 2 & 1 and Classes 3 & 2 appear generally for different values of (j,ℓ). However every Z_4 orbifold models with Wilson lines have conjugate pairs, R and $\overline{R}$, in the T_2 sector. These pairs lead to a flat direction in a way similar to the case without Wilson lines. This flat direction could break a gauge group reducing its rank by 6.

Here we discuss examples of models with Wilson lines. We take the Z_4 orbifold model with the following shift V^I;

$$V^I = \frac{1}{4}(2,1,1,0,\cdots,0)(0,\cdots,0). \tag{117}$$

In one example, we consider the above Z_4 orbifold model with the following Wilson line;

$$a^I_{e_2} = (0,\cdots,0)(1,0,\cdots,0). \tag{118}$$

This Wilson line is associated with the lattice vector e_2. This combination of V^I and $a^I_{e_2}$ is consistent with the modular invariance (114). We obtain $E_6\times SU(2)\times U(1)\times SO(16)'$ gauge group by V^I and $a^I_{e_2}$. This orbifold model has the following massless $\hat{U}_3$ fields;

$$(\overline{27},1;1)+(27,1;1)+(1,1;128_s). \tag{119}$$

The other untwisted subsectors, U_1 and U_2, are found as the fourth column for the # 3 shift in Table 2. Among the T_2 states, the following states;

$$|ie_1+e_4\rangle \pm |ie_1+e_3+e_4\rangle, \quad (i=0,1) \tag{120}$$

as well as $|ie_1 + ke_3\rangle$ $(i, k = 0, 1)$ have no Wilson line and their momenta are obtained as $P^I + 2V^I$. This shift corresponds to Classes 2 & 1. Thus the massless states of Eq.(120) have Eqs.(48), (49) and their "conjugates" as the shifted $E_8 \times E_8'$ momenta, leading to two conjugate pairs, R and $\overline{R}$. The other T_2 states;

$$|e_2 + ke_3\rangle \pm |e_1 + e_2 + ke_3\rangle, \quad |e_2 + e_4\rangle \pm |e_1 + e_2 + e_3 + e_4\rangle,$$
$$|e_2 + e_3 + e_4\rangle \pm |e_1 + e_2 + e_4\rangle, \tag{121}$$

have the momenta $P^I + 2V^I + a^I_{e_2}$. Note that $2V^I + a^I_{e_2}$ corresponds to Classes 2 & 3. These massless states (121) have Eq.(54) and their "conjugates" as the shifted $E_8 \times E_8'$ momenta, leading to four conjugate pairs, R and $\overline{R}$. Note that these four conjugate pairs have quantum numbers under $SO(16)'$. Therefore the $E_6 \times U(1)$ group is broken by VEVs of two conjugate pairs associated with Eq.(120) and its rank reduces by 2. The $SU(2)$ group also breaks. VEVs of four conjugate pairs corresponding to Eq.(121) reduce the rank of the $SO(16)'$ group by four. Further VEVs of the 27 and $\overline{27}$ ($128_s'$) in $\hat{U}_3$ reduce the rank of the $E_6 \times U(1)$ ($SO(16)'$) group by one.

We discuss another example. We consider the Z_4 orbifold model with above shift (117) and the following Wilson line;

$$a^I_{e_2} = (0, 1/2, 1/2, 0, \cdots, 0)(1/2, 1/2, 0, \cdots, 0). \tag{122}$$

This combination of V^I and $a^I_{e_2}$ is consistent with the modular invariance (114). This model has the $SO(10) \times SU(2) \times U(1)^2 \times E_7' \times SU(2)'$ gauge group. This model has the following $\hat{U}_3$;

$$(27, 1; 1, 1) + (\overline{27}, 1; 1, 1) + (1, 1; 56, 2) \tag{123}$$

where we use short notations 27 and $\overline{27}$ in place of $27 = 1 + 10 + 16$ and $\overline{27} = 1 + 10 + \overline{16}$, respectively. The states of Eqs.(120) have no Wilson lines and they have Eqs.(48), (49) and their "conjugates" as the $P^I + 2V^I$. The states of Eq.(121) have the momenta $P^I + 2V^I + a^I_{e_2}$, where $2V^I + a^I_{e_2}$ corresponds to Classes 3 & 2. The shifted momenta $P^I + 2V^I + a^I_{e_2}$ for massless states are written as Eq.(54) and their "conjugates", including four conjugate pairs. Thus VEVs of these conjugate pairs reduce the rank of $SO(10) \times U(1)^2$ by 6. Also $SU(2)$ and $SU(2)'$ are broken. VEVs of the $\hat{U}_3$ sector can break $SO(10) \times U(1)^2$ and E_7' reducing their ranks by one.

Similarly we can study T_n sectors of the other Z_{2n} orbifold models with nontrivial Wilson lines. Especially T_3 of Z_6-I and T_6 of Z_{12}-I and II orbifold models cannot have nontrivial Wilson lines as shown in Appendix A, although other sectors of these orbifold models have nontrivial Wilson lines. For these orbifold models, situation on flat directions is the same as the case without Wilson lines. In general, only Wilson lines with the order 2 are allowed for T_n sectors of the other Z_{2n} orbifold models as shown in Appendix A. Hence $nV^I + a^I_{e_i}$ is a shift

vector with the order 2. Its E_8 part is classifies into three classes, Classes 1, 2 and 3. The modular invariance allows only the two combinations, Classes 2 & 1 and 3 & 2. Thus massless matter fields of T_n are obtained by Eqs.(48), (49), (54) and their "conjugates". Conjugate pairs appear for the states by linear combination of the same content with values of $\tilde{\Delta}_n$ related by the conjugation in Table 1. VEVs of these pairs could lead to flat directions in a way similar to the case without Wilson lines. Therefore a drastic symmetry breaking could also happen in the presence of nontrivial Wilson lines.

4.4 Anomalies

Along flat directions, several fields become massive as seen in subsection 4.2. It is interesting to study flow of universal indices of models such as anomaly coefficients along these flat directions if any universal indices exist. In superstring theories, anomalies as mixed $U(1)$ anomalies and mixed target-space duality anomalies can be canceled by the Green-Schwarz mechanism [28]. This mechanism is independent of gauge groups. Thus these mixed anomaly coefficients should be universal for gauge groups, although target-space duality anomaly coefficients are not always universal for unrotated planes under some twists because of another group-dependent cancellation mechanism, i.e., threshold corrections due to massive modes [29, 30]. It is obvious that $U(1)$ anomaly coefficients do not change along flat directions if the anomalous $U(1)$ is not broken. Suppose that the anomalous $U(1)$ is unbroken by VEVs. Two fields in a mass term which is generated by flat directions have opposite anomalous $U(1)$ charges. Then integrating out these fields does not contribute to the anomalous $U(1)$ coefficients.

Now let us discuss the target-space duality anomaly coefficients b'^i_a for the i-th plane, which are obtained as

$$b'^i_a = -C(G_a) + \sum_{R_a} T(R_a)(1 + 2n^i_{R_a}), \tag{124}$$

where $C(G_a)$ and $T(R_a)$ are Casimir for the adjoint representation and the index for the R_a representation. Here $n^i_{R_a}$ is a modular weight of the state with R_a for the i-th moduli field [31]. For completely rotated planes, they should be independent of gauge groups, i.e. b'^i_a =constant (for a) so that the Green-Schwarz mechanism works [30]. This leads to a strong constraint on massless spectra, which is phenomenologically interesting [32, 33]. In Ref.[32], it is shown that flat directions along the U sectors do not change these duality anomaly coefficients if these U sectors correspond to completely rotated planes. Because these U sectors consist in $N = 4$ SUSY multiplets. It is notable that the T_n sectors of Z_{2n} orbifold models do not contribute to the duality anomaly (124) for completely rotated planes, because these sectors have $n^i = (-1/2, -1/2, 0)$ and the third plane is unrotated under θ^n. Hence integrating out these fields has no effect on the relation of duality anomaly coefficients. However, if other T_k sectors gain

masses by VEVs of T_n, integration of these T_k sectors changes Eq.(124). In general, b'^i_a are not universal for all unbroken gauge groups.

The flat directions are discussed in the presence of anomalous $U(1)$ symmetry ($U(1)_A$) [6, 7]. Its D-term is given as [34, 21],

$$D^A \equiv \frac{\delta^A_{GS}}{S+S^*} + q_\kappa |\Phi^\kappa|^2, \qquad (125)$$

where δ^A_{GS} is a coefficient of the Green-Schwarz mechanism to cancel the $U(1)_A$ anomaly [28] and q_κ is a $U(1)_A$ charge of a matter multiplet Φ^κ. The dilaton field S transforms nontrivially as $S \to S - i\delta^A_{GS}\theta(x)$ under $U(1)_A$ with the transformation parameter $\theta(x)$. This nontrivial transformation generates the contribution of S in the D-term (125). We have a relation $\langle S\rangle = 1/k_a g_a^2$ where g_a's are the gauge coupling constants for gauge groups G_a and k_a are corresponding Kac-Moody levels. The D-flatness of the anomalous $U(1)$ symmetry implies $\langle q_\kappa|\Phi^\kappa|^2\rangle \neq 0$ for a finite value of $\langle S\rangle$, although the flat directions we have discussed lead to $\langle q_\kappa|\Phi^\kappa|^2\rangle = 0$. Thus we need another type of VEVs leading to $\langle q_\kappa|\Phi^\kappa|^2\rangle \neq 0$, the other vanishing D-terms and F-flatness in order to obtain a finite value for $\langle S\rangle$. Within the presence of such VEVs, the D-flatness of $\langle R\rangle = \langle \bar{R}\rangle$, i.e., Eq. (65), is obvious. We can also investigate their F-flatness using explicit superpotentials. In general, one model has several combinations of VEVs leading to $\langle q_\kappa|\Phi^\kappa|^2\rangle \neq 0$, the other vanishing D-terms and F-flatness [6, 7, 10]. Some of such combinations might be inconsistent with the flat directions which we have discussed.

5 Conclusions and discussions

We have studied generic features related to matter contents and flat directions in Z_{2n} orbifold models. We have found the existence of model-independent conjugate pairs in the $\hat{U}_3$ and T_n sectors. We have classified a number of Z_{2n} orbifold models into only two types using the twisted sectors and the modular invariance.

Further applications of this classification are expected. For example, we can classify T_2 (T_4) sectors of Z_6 (Z_{12}) orbifold models in the similar way. These massless fields have the same $E_8 \times E_8'$ momenta as T_1 of Z_3 orbifold models. Further we can classify T_2 (T_3) sectors of Z_8 (Z_{12}) orbifold models.

Moreover these classification is very useful for $Z_{2n} \times Z_{2m}$ orbifold models with two independent twists, θ and ω. We can show that conjugate pairs, R and $\overline{R}$, appear in massless spectra of three twisted sectors, i.e., θ^n-twisted sector, ω^m-twisted sector and $\theta^n\omega^m$-twisted sector. These massless fields have the same shifted $E_8 \times E_8'$ momenta as the T_n sectors of Z_{2n} orbifold models. It is interesting to study flat directions for these sectors of $Z_{2n} \times Z_{2m}$ orbifold models in a way similar to the above.

It has been also shown that this classification is very useful for study on the breakings by flat directions. We have shown that there exist generic flat

directions in T_n and $\hat{U}_3$ sectors. Conjugate pairs, R and $\overline{R}$, lead to D-flatness as $\langle R\rangle = \langle\overline{R}\rangle \neq 0$. These VEVs, in general, lead to flat directions and break gauge groups drastically. These results are very important from phenomenological viewpoint. There can be another type of flat directions in other sectors, but we have not studied their features because the study requires a model-dependent analysis and is beyond the aim of this paper. The search for a realistic model based on Z_{2n} orbifold models has been worth challenging and our results should be extended further.

Other string models, e.g., by Calabi-Yau construction or fermionic construction include some conjugate pairs, R and $\overline{R}$. It is interesting to apply the above analyses to these models.

Much work is recently devoted to derive soft SUSY breaking terms from superstring theory [35, 36, 37]. In these aspects, study on flat directions are also important. For example, SUSY breaking in string models could lead to imaginary soft scalar masses at the Planck scale. Such initial conditions at the Planck scale are obtained in the case where moduli fields contribute on the SUSY breaking in a sizable way [36]. Imaginary soft scalar masses are also obtained in string models with anomalous $U(1)$ symmetries, even though the dilaton field breaks dominantly the SUSY [37]. Such negative mass squared terms along flat directions have important implications. Further SUSY breaking effects in supergravity might lift some flat directions and fix magnitudes of VEVs of matter multiplets determining gauge symmetry breaking pattern. In the case that there are flat directions in the SUSY limit, study of the structure of soft SUSY breaking terms and its phenomenological implications will be discussed elsewhere [38].

Other nonperturbative effects could lift these flat directions. In Ref.[39], it is shown nonperturbative effects like e^{-aS} are forbidden in some Calabi-Yau models with an anomalous $U(1)$ symmetry due to holomorphy of W and discrete symmetries. It is important to apply such analyses to the generic flat directions obtained in this paper.

Appendix A

Here we summarize the structure of T_n sectors of Z_{2n} orbifold models. In (a), v^i and the Lie lattice realizing it are shown. This lattice has the most degrees of freedom for Wilson lines among the Lie lattices leading to the same value of v^i. Transformation of vectors by θ is written in (b) **. In the following lattices, fixed points of T_n sectors are $(\theta^2, ie_1 + je_2 + ke_3 + \ell e_4)$ $(i,j,k,\ell = 0,1)$. Eigenstates of T_n are given in (c), where θ^n is omitted ††. Constraints on Wilson lines are written in (d), where a_b denotes $a^I_{e_b}$ and $Ma_b \approx 0$ means $Ma_b = 0$ (mod $\Gamma_{E_8\times E_8'}$).

** For the Z_4 orbifold model, (b) is omitted. See Eqs. (4) and (5).

††Eigenstates written in Ref.[12] include a minor mistake. That should be replaced.

A.1 Z_4 orbifold model

(a) $v^i = 1/4(1,1,-2)$ $SO(5)^2 \times SU(2)^2$ lattice

(c) Eigenstates

$$\begin{gathered} |ie_1 + ke_3\rangle \\ |e_2 + ke_3\rangle \pm |e_1 + e_2 + ke_3\rangle, \quad |ie_1 + e_4\rangle \pm |ie_1 + e_3 + e_4\rangle, \\ |e_2 + e_4\rangle \pm |e_1 + e_2 + e_3 + e_4\rangle, \quad |e_2 + e_3 + e_4\rangle \pm |e_1 + e_2 + e_4\rangle. \end{gathered}$$

(d) Wilson lines $2a_2 \approx 2a_4 \approx 2a_5 \approx 2a_6 \approx 0$, $(a_1 \approx a_3 \approx 0)$.

A.2 Z_6-I orbifold model

(a) $v^i = 1/6(1,1,-2)$ $G_2^2 \times SU(3)$ lattice

(b) Twist

$$\begin{gathered} \theta e_a = 2e_a + 3e_{a+1}, \quad \theta e_{a+1} = -e_a - e_{a+1}, \quad a = 1,3, \\ \theta e_5 = e_6, \quad \theta e_6 = -e_5 - e_6. \end{gathered}$$

(c) Eigenstates

$$\begin{gathered} |0\rangle, \quad |e_a\rangle + \alpha|e_a + e_{a+1}\rangle + \alpha^2|e_{a+1}\rangle, \quad (a = 1,3), \\ |e_1 + e_3\rangle + \alpha|e_1 + e_2 + e_3 + e_4\rangle + \alpha^2|e_2 + e_4\rangle, \\ |e_1 + e_3 + e_4\rangle + \alpha|e_1 + e_2 + e_4\rangle + \alpha^2|e_2 + e_3\rangle, \\ |e_1 + e_4\rangle + \alpha|e_1 + e_2 + e_3\rangle + \alpha^2|e_2 + e_3 + e_4\rangle, \end{gathered}$$

where $\alpha = \exp[2\pi i m/3]$ and $m = 0,1,2$.

(d) Wilson lines $3a_5 \approx 0$, $(a_5 \approx a_6)$, $a_b \approx 0$ $(b = 1 \sim 4)$.

A.3 Z_6-II orbifold model

(a) $v^i = 1/6(1,-3,2)$ $G_2 \times SU(2)^2 \times SU(3)$ lattice

(b) Twist

$$\begin{gathered} \theta e_1 = 2e_1 + 3e_2, \quad \theta e_2 = -e_1 - e_2, \quad \theta e_a = -e_a, \quad a = 3,4 \\ \theta e_5 = e_6, \quad \theta e_6 = -e_5 - e_6. \end{gathered}$$

(c) Eigenstates

$$|ke_3 + \ell e_4\rangle,$$
$$|e_1 + ke_3 + \ell e_4\rangle + \alpha|e_1 + e_2 + ke_3 + \ell e_4\rangle + \alpha^2|e_2 + ke_3 + \ell e_4\rangle,$$

where $\alpha = \exp[2\pi i m/3]$ and $m = 0, 1, 2$.

(d) Wilson lines $2a_3 \approx 2a_4 \approx 3a_5 \approx 0$, $(a_5 \approx a_6,\ \ a_1 \approx a_2 \approx 0)$.

A.4 Z_8-I orbifold model

(a) $v^i = 1/8(1, -3, 2)$ $\qquad SO(9) \times SO(5)$ lattice

(b) Twist

$$\theta e_1 = e_2, \quad \theta e_2 = e_3, \quad \theta e_3 = e_1 + e_2 + e_3 + 2e_4,$$
$$\theta e_4 = -\sum_{a=1}^4 e_a, \quad \theta e_5 = e_5 + 2e_6, \quad \theta e_6 = -e_5 - e_6.$$

(c) Eigenstates

$$|i(e_1 + e_3)\rangle, \quad |e_1 + e_2\rangle \pm |e_2 + e_3\rangle,$$
$$|e_1\rangle + \alpha|e_1 + e_2 + e_3\rangle + \alpha^2|e_3\rangle + \alpha^3|e_2\rangle,$$
$$|e_4\rangle + \alpha|e_3 + e_4\rangle + \alpha^2|e_2 + e_3 + e_4\rangle + \alpha^3|\sum_{a=1}^4 e_a\rangle,$$
$$|e_1 + e_4\rangle + \alpha|e_1 + e_2 + e_4\rangle + \alpha^2|e_2 + e_4\rangle + \alpha^3|e_1 + e_3 + e_4\rangle,$$

where $\alpha = \exp[\pi i m/2]$ and $m = 0, 1, 2, 3$.

(d) Wilson lines $2a_4 \approx 2a_6 \approx 0$, $(a_1 \approx a_2 \approx a_3 \approx a_5 \approx 0)$.

A.5 Z_8-II orbifold model

(a) $v^i = 1/8(1, 3, -4)$ $\qquad SO(9) \times SU(2)^2$ lattice

(b) Twist

$$\theta e_1 = e_2, \quad \theta e_2 = e_3, \quad \theta e_3 = e_1 + e_2 + e_3 + 2e_4,$$
$$\theta e_4 = -\sum_{a=1}^4 e_a, \quad \theta e_b = -e_b, \quad b = 5, 6.$$

(c) Eigenstates are same those of T_4 for the Z_8-I orbifold models.

(d) Wilson lines $2a_4 \approx 2a_5 \approx 2a_6 \approx 0$, $(a_1 \approx a_2 \approx a_3 \approx 0)$.

A.6 Z_{12}-I orbifold model

(a) $v^i = 1/12(1, -5, 4)$ $\quad F_4 \times SU(3)$ lattice

(b) Twist

$$\theta e_2 = \sum_{b=1}^{3} e_b, \quad \theta e_a = e_{a+1}, \quad a = 1, 3, 5,$$
$$\theta e_4 = -2e_1 - 2e_2 - e_3 - e_4, \quad \theta e_5 = -e_5 - e_6.$$

(c) Eigenstates

$$\begin{gathered}
|0\rangle, \quad |e_3\rangle + \alpha|e_3 + e_4\rangle + \alpha^2|e_4\rangle, \\
|e_1\rangle + \beta|\textstyle\sum_{a=1}^{4} e_a\rangle + \beta^2|e_2 + e_3\rangle \\
+\beta^3|e_1 + e_3 + e_4\rangle + \beta^4|e_1 + e_2 + e_3\rangle + \beta^5|e_2\rangle, \\
|e_1 + e_2\rangle + \beta|e_2 + e_3 + e_4\rangle + \beta^2|e_1 + e_4\rangle \\
+\beta^3|e_1 + e_2 + e_4\rangle + \beta^4|e_2 + e_4\rangle + \beta^5|e_1 + e_3\rangle,
\end{gathered}$$

where $\alpha = \exp[2\pi i m/3]$ and $m = 0, 1, 2$ and $\beta = \exp[\pi i k/3]$ and $k = 0, 1, \cdots, 5$.

(d) Wilson lines $3a_5 \approx 0$, $(a_5 \approx a_6,\ a_b \approx 0\ \ (b = 1 \sim 4))$.

A.7 Z_{12}-II orbifold model

(a) $v^i = 1/12(1, 5, -6)$ $\quad F_4 \times SU(2)^2$ lattice

(b) Twist

$$\theta e_a = -e_a, \quad a = 5, 6.$$

The others e_a $(a = 1 \sim 4)$ are transformed in the same way as those for T_6 of the Z_{12}-I orbifold models.

(c) Eigenstates are same those of T_4 for the Z_{12}-I orbifold models.

(d) Wilson lines $2a_5 \approx 2a_6 \approx 0$, $(a_b \approx 0\ \ (b = 1 \sim 4))$.

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Table Captions

Table 1 Conjugation of $\tilde{\Delta}_n$ and degeneracy factors. The second column shows the conjugation of $\tilde{\Delta}_n$ under $P + nV \to -(P + nV)$. In the third column, the common number of degeneracy factors are shown for $\tilde{\Delta}_n = \omega_{(n)}^k$ and $\tilde{\Delta}_n = \omega_{(n)}^{-k-\delta}$.

Table 2 Shifts for Z_4 orbifold models. The second column shows the independent E_8 shifts. The third column shows the gauge group. In the fourth and fifth columns, untwisted matters are shown in U_1 & U_2 and U_3 subsectors, respectively.

Table 3 Shifts for Z_6 orbifold models. The second column shows the independent E_8 shifts. The third column shows the gauge group. In the fourth, fifth and sixth columns, untwisted matters are shown in $6PV = 1$, $6PV = 2$ and $6PV = 3$ subsectors, respectively.

Table 4 Z_4 orbifold models. The second column shows gauge groups omitting $U(1)$ groups. The third column shows combinations of E_8 shifts and E_8' shifts as the corresponding numbers of the first column in Table 2. Massless states of T_1 and T_2 are shown in the fourth and fifth columns.

Table 5 T_3 of Z_6-I orbifold models. The third column shows combinations of E_8 shifts and E_8' shifts as the corresponding numbers of the first column in Table 3. Massless states of T_3 are shown in the fourth column. In the column of gauge group, $G_{6,3,2}$ denotes $SU(6) \times SU(3) \times SU(2)$ and $U(1)$ groups are omitted.

Table 6 T_3 of Z_6-II orbifold models. The third column shows combinations of E_8 shifts and E_8' shifts as the corresponding numbers of the first column in Table 3. Massless states of T_3 are shown in the fourth column. In the column of gauge group, $G_{6,3,2}$ denotes $SU(6) \times SU(3) \times SU(2)$ and $U(1)$ groups are omitted.

Table 1. conjugation of $\tilde{\Delta}_n$ and degeneracy factors.

Orbifold	conjugation of $\tilde{\Delta}_n$	degeneracy
Z_4	$\tilde{\Delta}_2 = 1 \longleftrightarrow \tilde{\Delta}_2 = -1$	6
Z_6-I	$\tilde{\Delta}_3 = e^{-2\pi i/3}$ invariant	5
	$\tilde{\Delta}_3 = 1 \longleftrightarrow \Delta_3 = e^{2\pi i/3}$	5
Z_6-II	$\tilde{\Delta}_3 = e^{2\pi i/3}$ invariant	4
	$\tilde{\Delta}_3 = 1 \longleftrightarrow \Delta_3 = e^{-2\pi i/3}$	4
Z_8-I	$\tilde{\Delta}_4 = 1 \longleftrightarrow \tilde{\Delta}_4 = i$	3
	$\tilde{\Delta}_4 = -1 \longleftrightarrow \Delta_4 = -i$	3
Z_8-II	$\tilde{\Delta}_4 = 1 \longleftrightarrow \Delta_4 = -1$	4
	$\tilde{\Delta}_4 = \pm i$ invariant	3
Z_{12}-I	$\tilde{\Delta}_6 = 1 \longleftrightarrow \tilde{\Delta}_6 = e^{2\pi i/3}$	3
	$\tilde{\Delta}_6 = e^{-2\pi i/3}$ invariant	3
	$\tilde{\Delta}_6 = -1 \longleftrightarrow \Delta_6 = e^{-\pi i/3}$	2
	$\tilde{\Delta}_6 = e^{\pi i/3}$ invariant	2
Z_{12}-II	$\tilde{\Delta}_6 = e^{m\pi i/3} \longleftrightarrow \tilde{\Delta}_6 = -e^{m\pi i/3}$	2

Table 2. Shifts for Z_4 orbifold models

#	Shift ($4V^I$)	Gauge group	U_1, U_2	U_3
0	(00000000)	E_8		
1	(22000000)	$E_7 \cdot SU_2$		(56,2)
2	(11000000)	$E_7 \cdot U_1$	(56)	2(1)
3	(21100000)	$E_6 \cdot SU_2 \cdot U_1$	$(\overline{27}, 2) + (1, 2)$	$(\overline{27}, 1) + (27, 1)$
4	(40000000)	SO_{16}		(128_s)
5	(20000000)	$SO_{14} \cdot U_1$	(64_s)	$2(14_v)$
6	(31000000)	$SO_{12} \cdot SU_2 \cdot U_1$	$(32_c, 1) + (12_v, 2)$	$(32_c, 1) + 2(1, 1)$
7	(22200000)	$SO_{10} \cdot SU_4$	$(16_c, 4)$	$(10_v, 6)$
8	(31111100)	$SU_8 \cdot SU_2$	$(28, 2)$	$(70, 1)$
9	(1111111-1)	$SU_8 \cdot U_1$	$(\overline{56}) + (8)$	$(28) + (\overline{28})$

Table 3. Shifts for Z_6 orbifold models

#	Shift ($4V^I$)	Gauge group	$U(6PV=1)$	$U(6PV=2)$	$U(6PV=3)$
0	(00000000)	E_8			
1	(33000000)	$E_7 \cdot SU_2$			(56,2)
2	(11000000)	$E_7 \cdot U_1$	56	1	
3	(22000000)	$E_7 \cdot U_1$		56+1	
4	(42200000)	$E_6 \cdot SU_3$		$(\overline{27},\overline{3})$	
5	(21100000)	$E_6 \cdot SU_2 \cdot U_1$	$(\overline{27},2)$	$(27,1)$	$2(1,2)$
6	(32100000)	$E_6 \cdot U_1^2$	$\overline{27}+1+1$	$\overline{27}+1$	$27+\overline{27}$
7	(60000000)	SO_{16}			128_s
8	(20000000)	$SO_{14} \cdot U_1$	64_c	14_v	
9	(40000000)	$SO_{14} \cdot U_1$		64_c+14_v	
10	(42000000)	$SO_{12} \cdot SU_2 \cdot U_1$	$(32_s,1)$	$(12_v,2)+(1,1)$	$(32_c,2)$
11	(51000000)	$SO_{12} \cdot SU_2 \cdot U_1$	$(12_v,2)$	$(32_s,1)+(1,1)$	$(32_c,2)$
12	(31000000)	$SO_{12} \cdot U_1^2$	32_s+12_v	32_c+1+1	12_v+12_v
13	(22200000)	$SO_{10} \cdot SU_3 \cdot U_1$	$(16_s,\overline{3})$	$(10_v,3)+(1,3)$	$(16_s,1)+(16_c,1)$
14	(33200000)	$SO_{10} \cdot SU_2^2 \cdot U_1$	$(16_s,1,2)$ $+(1,2,2)$	$(16_s,2,1)$ $+(10_v,1,1)$	$(10_v,2,2)$
15	(41100000)	$SO_{10} \cdot SU_2 \cdot U_1^2$	$(16_c,1)+(10_v,2)$ $+(1,2)$	$(16_s,2)+(10_v,1)$ $+(1,1)$	$(16_c,2)+(16_s,1)$ +2(1,2)
16	(51110000)	$SO_8 \cdot SU_4 \cdot U_1$	$(8_c,4)+(8_v,1)$	$(8_s,4)+(1,6)$	$(8_v,6)$
17	(51111111)	SU_9		84	
18	(1111111-1)	$SU_8 \cdot U_1$	$\overline{56}$	28	$8+\overline{8}$
19	(5111111-1)	$SU_8 \cdot U_1$	$\overline{28}+1$	$\overline{28}$	70
20	(7111111-1)/2	$SU_7 \cdot SU_2 \cdot U_1$	$(21,2)$	$(\overline{35},1)+(\overline{7},1)$	$(7,2)+(\overline{7},2)$
21	(31111111)	$SU_7 \cdot U_1^2$	$35+\overline{7}+1$	$21+\overline{7}+\overline{7}$	$21+\overline{21}$
22	(91111111)/2	$SU_7 \cdot U_1^2$	$21+7+\overline{7}$	$35+7+1$	$21+\overline{21}$
23	(51111100)	$SU_6 \cdot SU_3 \cdot SU_2$	$(\overline{6},\overline{3},2)$	$(\overline{15},3,1)$	$(20,1,2)$
24	(93311111)/2	$SU_6 \cdot SU_3 \cdot U_1$	$(20,1)+(\overline{6},\overline{3})$	$(15,\overline{3})+(1,1)$	$(6,3)+(\overline{6},\overline{3})$
25	(3311111-1)	$SU_6 \cdot SU_2^2 \cdot U_1$	$(\overline{15},1,2)$ $+(6,2,1)$	$(15,1,1)$ $+(\overline{6},2,2)$	$(20,2,1)$ $+2(1,1,2)$
26	(22222000)	$SU_5 \cdot SU_4 \cdot U_1$	$(\overline{10},4)+(1,\overline{4})$	$(\overline{10},1)+(5,6)$	$(5,4)+(\overline{5},\overline{4})$

Table 4. Z_4 orbifold models

No.	Gauge group	V^I	T_1	T_2
1	$E_6 \cdot SU_2 \cdot E_8'$	3;0	$16(\overline{27},1,1)+32(1,2;1)$ $+80(1,1;1)$	$10(\overline{27},1;1)+6(27,1;1)$ $+32(1,2;1)+16(1,1;1)$
2	$E_6 \cdot SU_2 \cdot E_7' \cdot SU_2'$	3;1	$16(1,2;1,2)+32(1,2;1,2)$	$10(27,1;1,1)+6(\overline{27},1;1,1)$ $+32(1,2;1,1)+16(1,1;1,1)$
3	$E_6 \cdot SU_2 \cdot SO_{16}'$	3;4	$16(1,1;16_v)$	$10(\overline{27},1;1)+6(27,1;1)$ $+32(1,2;1)+16(1,1;1)$
4	$SO_{14} \cdot E_7'$	5;2	$16(14_v;1)+96(1;1)$	$16(14_v;1)+32(1;1)$
5	$SO_{14} \cdot SO_{12}' \cdot SU_2'$	5;6	$16(1;12_v,1)+32(1;1,2)$	$16(14_v;1,1)+32(1;1,1)$
6	$SO_{10} \cdot SU_4 \cdot E_7'$	7;2	$16(16_s,1;1)+32(1,4;1)$	$16(10_v,1;1)+16(1,6;1)$
7	$SO_{10} \cdot SU_4 \cdot SO_{12}' \cdot SU_2'$	7;6	$16(1,4;1,2)$	$16(10_v,1;1,1)+16(1,6;1,1)$
8	$SU_8 \cdot SU_2 \cdot E_8'$	8;0	$16(8,2;1)+32(\overline{8},1,;1)$	$10(\overline{28},1;1)+6(28,1;1)$ $+32(1,2;1)$
9	$SU_8 \cdot SU_2 \cdot E_7' \cdot SU_2'$	8;1	$16(\overline{8},1;1,2)$	$10(28,1;1,1)+6(\overline{28},1;1,1)$ $+32(1,2;1,1)$
10	$SU_8 \cdot SU_2 \cdot SO_{16}'$	8;4		$10(\overline{28},1;1)+6(28,1;1)$ $+32(1,2;1)$
11	$SU_8 \cdot E_6' \cdot SU_2'$	9;3	$16(8;1,1)+16(1;1,2)$ $+32(1;1,1)$	$10(\overline{8};1,2)+6(8;1,2)$
12	$SU_8 \cdot SU_8' \cdot SU_2'$	9;8	$16(1;\overline{8},1)$	$10(8;1,2)+6(\overline{8};1,2)$

Table 5-1. T_3 of Z_6-I orbifold models

No.	Gauge group	V^I	T_3
1	$E_7 \cdot SU_2 \cdot E_8'$	1;0	$5(56,1;1)+22(1,2;1)$
2	$E_7 \cdot SU_2 \cdot E_6' \cdot SU_3'$	1;4	$5(56,1;1,1)+22(1,2;1,1)$
3	$E_6 \cdot SU_2 \cdot E_8'$	5;0	$5(27,1;1)+6(\overline{27},1;1)+22(1,2;1)+10(1,1;1)$
4	$E_6 \cdot SU_2 \cdot E_6' \cdot SU_3'$	5;4	$5(27,1;1,1)+6(\overline{27},1;1,1)+22(1,2;1,1)+10(1,1;1,1)$
5	$SO_{16} \cdot E_7' \cdot SU_2'$	7;1	$5(16_v;1,2)$
6	$SO_{16} \cdot E_6' \cdot SU_2'$	7;5	$5(16_v;1,2)$
7	$SO_{14} \cdot E_7'$	8;2	$11(14_v;1)+21(1;1)$
8	$SO_{14} \cdot E_6'$	8;6	$11(14_v;1)+21(1;1)$
9	$E_7 \cdot SO_{14}'$	2;9	$5(56;1)+42(1;1)$
10	$E_6 \cdot SO_{14}'$	6;9	$6(27;1)+5(\overline{27};1)+52(1;1)$
11	$SO_{14} \cdot SO_{12}' \cdot SU_2'$	8;11	$5(14_v;1,2)+11(1;1,2)$
12	$SO_{12} \cdot SU_2 \cdot SO_{14}'$	11;9	$5(32_c,1;1)+11(12_v,1;1)+22(1,2;1)$
13	$SO_{12} \cdot E_7'$	12;3	$5(32_s;1)+11(12_v;1)+42(1;1)$
14	$SO_{12} \cdot SU_2 \cdot SO_{12}'$	10;12	$11(12_v,1;1)+21(1,2;1)$
15	$SO_{10} \cdot SU_3 \cdot E_7' \cdot SU_2'$	13;1	$5(10_v,1;1,2)+5(1,3;1,2)+6(1,\overline{3};1,2)$
16	$SO_{10} \cdot SU_3 \cdot E_6' \cdot SU_2'$	13;5	$5(10_v,1;1,2)+5(1,3;1,2)+6(1,\overline{3};1,2)$
17	$SO_{10} \cdot SU_2^2 \cdot E_7'$	14;3	$6(16_s,1,1;1)+5(16_c,1,1;1)+5(10_v,2,1;1)$ $+11(1,2,1;1)+22(1,1,2;1)$
18	$SO_{12} \cdot SU_2 \cdot SO_{10}' \cdot SU_2'^2$	10;14	$5(12_v,1;1,1,2)+11(1,2;1,1,2)$
19	$SO_{10} \cdot SU_2 \cdot E_8'$	15;0	$6(16_s,1;1)+5(16_c,1;1)+5(10_v,2;1)$ $+11(1,2;1)+42(1,1;1)$
20	$SO_{10} \cdot SU_2 \cdot E_6' \cdot SU_3'$	15;4	$6(16_s,1;1,1)+5(16_c,1;1,1)+5(10_v,2;1,1)$ $+11(1,2;1,1)+42(1,1;1,1)$
21	$SO_{16} \cdot SO_{10}' \cdot SU_2'$	7;15	$11(16_v;1,1)$
22	$SO_{10} \cdot SU_3 \cdot SO_{10}' \cdot SU_2'$	13;15	$11(10_v,1;1,1)+11(1,3;1,1)+10(1,\overline{3};1,1)$
23	$SO_8 \cdot SU_4 \cdot E_7'$	16;2	$11(8_s,1;1)+10(1,4;1)+11(1,\overline{4};1)$
24	$SO_8 \cdot SU_4 \cdot E_6'$	16;6	$11(8_s,1;1)+10(1,4;1)+11(1,\overline{4};1)$
25	$SO_8 \cdot SU_4 \cdot SO_{12}' \cdot SU_2'$	16;11	$5(8_s,1;1,2)+6(1,4;1,2)+5(1,\overline{4};1,2)$
26	$SO_{12} \cdot SU_9'$	12;17	$5(32_s;1)+11(12_v;1)+42(1;1)$
27	$SO_{10} \cdot SU_2^2 \cdot SU_9'$	14;17	$6(16_s,1,1;1)+5(16_c,1,1;1)+5(10_v,2,1;1)$ $+11(1,2,1;1)+22(1,1,2;1)$
28	$SU_8 \cdot SO_{12}'$	18;12	$10(8;1)+11(\overline{8};1)$

Table 5-2. T_3 of Z_6-I orbifold models

No.	Gauge group	V^I	T_3
29	$SU_8 \cdot SO'_{10} \cdot SU'^2_2$	18;14	$6(8;1,1,2)+5(\overline{8};1,1,2)$
30	$SU_8 \cdot SO'_{12}$	19;12	$11(8;1)+10(\overline{8};1)$
31	$SU_8 \cdot SO'_{10} \cdot SU'^2_2$	19;14	$5(8;1,1,2)+6(\overline{8};1,1,2)$
32	$SO_{14} \cdot SU'_7 \cdot SU'_2$	8;20	$5(14_v;1,2)+11(1;1,2)$
33	$SU_7 \cdot SU_2 \cdot SO'_{14}$	20;9	$6(21,1;1)+5(\overline{21},1;1)+5(7,1;1)$ $+5(\overline{7},1;1)+22(1,2;1)$
34	$SO_8 \cdot SU_4 \cdot SU'_7 \cdot SU'_2$	16;20	$5(8_s,1;1,2)+6(1,4;1,2)+5(1,\overline{4};1,2)$
35	$SU_7 \cdot E'_7$	21;2	$10(7;1)+11(\overline{7};1)+22(1;1)$
36	$SU_7 \cdot E'_6$	21;6	$10(7;1)+11(\overline{7};1)+22(1;1)$
37	$SU_7 \cdot SO'_{12} \cdot SU'_2$	21;11	$6(7;1,2)+5(\overline{7};1,2)+10(1;1,2)$
38	$SU_7 \cdot SU'_7 \cdot SU'_2$	21;20	$6(7;1,2)+5(\overline{7};1,2)+10(1;1,2)$
39	$SU_7 \cdot E'_7$	22;3	$5(21;1)+6(\overline{21};1)+5(7;1)+5(\overline{7},1)+42(1;1)$
40	$SO_{12} \cdot SU_2 \cdot SU'_7$	10;22	$11(12_v,1;1)+21(1,2;1)$
41	$SU_7 \cdot SU'_9$	22;17	$5(21;1)+6(\overline{21};1)+5(7;1)+5(\overline{7};1)+42(1;1)$
42	$SU_8 \cdot SU'_7$	18;22	$10(8;1)+11(\overline{8};1)$
43	$SU_8 \cdot SU'_7$	19;22	$11(8;1)+10(\overline{8};1)$
44	$G_{6,3,2} \cdot E'_8$	23;0	$5(20,1,1;1)+5(6,3,1;1)+6(\overline{6},\overline{3},1;1)+22(1,1,2;1)$
45	$G_{6,3,2} \cdot E'_6 \cdot SU'_3$	23;4	$5(20,1,1;1,1)+5(6,3,1;1,1)+6(\overline{6},\overline{3},1;1,1)+22(1,1,2;1,1)$
46	$SO_{16} \cdot G'_{6,3,2}$	7;23	$5(16_v;1,1,2)$
47	$SO_{10} \cdot SU_3 \cdot G'_{6,3,2}$	13;23	$5(10_v,1;1,1,2)+5(1,3;1,1,2)+6(1,\overline{3};1,1,2)$
48	$SO_{14} \cdot SU'_6 \cdot SU'_3$	8;24	$11(14_v;1,1)+21(1;1,1)$
49	$SU_6 \cdot SU_3 \cdot SO'_{14}$	24;9	$5(20,1;1)+6(6,3;1)+5(\overline{6},\overline{3};1)+42(1,1;1)$
50	$SO_8 \cdot SU_4 \cdot SU'_6 \cdot SU'_3$	16;24	$11(8_s,1;1,1)+10(1,4;1,1)+11(1,\overline{4};1,1)$
51	$SU_7 \cdot SU'_6 \cdot SU'_3$	21;24	$10(7;1,1)+11(\overline{7};1,1)+22(1;1,1)$
52	$SU_6 \cdot SU^2_2 \cdot E'_7 \cdot SU'_2$	25;1	$6(6,1,1;1,2)+5(\overline{6},1,1;1,2)+5(1,2,2;1,2)$
53	$SU_6 \cdot SU^2_2 \cdot E'_6 \cdot SU'_2$	25;5	$6(6,1,1;1,2)+5(\overline{6},1,1;1,2)+5(1,2,2;1,2)$
54	$SU_6 \cdot SU^2_2 \cdot SO'_{10} \cdot SU'_2$	25;15	$10(6,1,1;1,1)+11(\overline{6},1,1;1,1)+11(1,2,2;1,1)$
55	$SU_6 \cdot SU^2_2 \cdot G'_{6,3,2}$	25;23	$6(6,1,1;1,1,2)+5(\overline{6},1,1;1,1,2)+5(1,2,2;1,1,2)$
56	$SU_5 \cdot SU_4 \cdot SO'_{12}$	26;12	$11(5,1;1)+10(\overline{5},1;1)+11(1,6;1)$
57	$SU_5 \cdot SU_4 \cdot SO'_{10} \cdot SU'^2_2$	26;14	$5(5,1;1,1,2)+6(\overline{5},1;1,1,2)+5(1,6;1,1,2)$
58	$SU_5 \cdot SU_4 \cdot SU'_7$	26;22	$11(5,1;1)+10(\overline{5},1;1)+11(1,6;1)$

Table 6-1. T_3 of Z_6-II orbifold models

No.	Gauge group	V^I	T_3
1	$E_7 \cdot E_8'$	2;0	$4(56;1)+44(1;1)$
2	$E_7 \cdot SU_2 \cdot E_7'$	1;3	$4(56,1;1)+20(1,2;1)$
3	$E_7 \cdot E_6' \cdot SU_3'$	2;4	$4(56,1;1)+44(1,1;1)$
4	$E_6 \cdot SU_2 \cdot E_7'$	5;3	$8(27,1;1)+4(\overline{27},1;1)+20(1,2;1)+8(1,1;1)$
5	$E_6 \cdot E_8'$	6;0	$4(27;1)+8(\overline{27};1)+52(1;1)$
6	$E_6 \cdot E_6' \cdot SU_3'$	6;4	$4(27;1,1)+8(\overline{27};1,1)+52(1;1,1)$
7	$SO_{16} \cdot E_7'$	7;2	$12(16_v;1)$
8	$SO_{16} \cdot E_6'$	7;6	$12(16_v;1)$
9	$SO_{12} \cdot SU_2 \cdot E_7' \cdot SU_2'$	10;1	$4(12_v,1;1,2)+12(1,2;1,2)$
10	$SO_{12} \cdot SU_2 \cdot E_6' \cdot SU_2'$	10;5	$4(12_v,1;1,2)+12(1,2;1,2)$
11	$SO_{12} \cdot SU_2 \cdot E_8'$	11;0	$4(32_c,1;1)+12(12_v,1;1)+20(1,2;1)$
12	$SO_{12} \cdot SU_2 \cdot E_6' \cdot SU_3'$	11;4	$4(32_c,1;1,1)+12(12_v,1;1,1)+20(1,2;1,1)$
13	$SO_{16} \cdot SO_{12} \cdot SU_2'$	7;11	$4(16_v;1,2)$
14	$SO_{14} \cdot SO_{12}'$	8;12	$12(14_v;1)+20(1;1)$
15	$SO_{12} \cdot SO_{14}'$	12;9	$4(32_s;1)+12(12_v;1)+44(1;1)$
16	$SO_{10} \cdot SU_3 \cdot E_7'$	13;2	$12(10_v,1;1)+8(1,3;1)+12(1,\overline{3};1)$
17	$SO_{10} \cdot SU_3 \cdot E_6'$	13;6	$12(10_v,1;1)+8(1,3;1)+12(1,\overline{3};1)$
18	$SO_{10} \cdot SU_3 \cdot SO_{12}' \cdot SU_2'$	13;11	$4(10_v,1;1,2)+8(1,3;1,2)+4(1,\overline{3};1,2)$
19	$SO_{14} \cdot SO_{10}' \cdot SU_2'^2$	8;14	$4(14_v;1,1,2)+12(1;1,1,2)$
20	$SO_{10} \cdot SU_2^2 \cdot SO_{14}'$	14;9	$4(16_s,1,1;1)+8(16_c,1,1;1)+4(10_v,2,1;1)$ $+12(1,2,1;1)+20(1,1,2;1)$
21	$SO_{10} \cdot SU_2 \cdot E_7'$	15;3	$4(16_s,1;1)+8(16_c,1;1)+4(10_v,2;1)$ $+12(1,2;1)+44(1,1;1)$
22	$SO_{12} \cdot SU_2 \cdot SO_{10}' \cdot SU_2'$	10;15	$12(12_v,1;1,1)+20(1,2;1,1)$
23	$SO_8 \cdot SU_4 \cdot SO_{12}'$	16;12	$12(8_s,1;1)+12(1,4;1)+8(1,\overline{4};1)$
24	$SO_8 \cdot SU_4 \cdot SO_{10}' \cdot SU_2'^2$	16;14	$4(8_s,1;1,1,2)+4(1,4;1,1,2)+8(1,\overline{4};1,1,2)$
25	$E_7 \cdot SU_2 \cdot SU_9'$	1;17	$4(56,1;1)+20(1,2;1)$
26	$E_6 \cdot SU_2 \cdot SU_9'$	5;17	$8(27,1;1)+4(\overline{27},1;1)+20(1,2;1)+8(1,1;1)$
27	$SO_{10} \cdot SU_2 \cdot SU_9'$	15;17	$4(16_s,1;1)+8(16_c,1;1)+4(10_v,2;1)$ $+12(1,2;1)+44(1,1;1)$
28	$SU_8 \cdot E_7' \cdot SU_2'$	18;1	$4(8;1,2)+8(\overline{8};1,2)$
29	$SU_8 \cdot E_6' \cdot SU_2'$	18;5	$4(8;1,2)+8(\overline{8};1,2)$
30	$SU_8 \cdot SO_{10}' \cdot SU_2'$	18;15	$12(8;1,1)+8(\overline{8};1,1)$

Table 6-2. T_3 of Z_6-II orbifold models

No.	Gauge group	V^I	T_3
31	$SU_8 \cdot E_7' \cdot SU_2'$	19;1	$8(8;1,2)+4(\overline{8};1,2)$
32	$SU_8 \cdot E_6' \cdot SU_2'$	19;5	$8(8;1,2)+4(\overline{8};1,2)$
33	$SU_8 \cdot SO_{10}' \cdot SU_2'$	19;15	$8(8;1,1)+12(\overline{8};1,1)$
34	$SU_7 \cdot SU_2 \cdot E_8'$	20;0	$4(21,1;1)+8(\overline{21},1;1)+4(7,1;1)$ $+4(\overline{7},1;1)+20(1,2;1)$
35	$SU_7 \cdot SU_2 \cdot E_6' \cdot SU_3'$	20;4	$4(21,1;1,1)+8(\overline{21},1;1,1)+4(7,1;1,1)$ $+4(\overline{7},1;1,1)+20(1,2;1,1)$
36	$SO_{16} \cdot SU_7' \cdot SU_2'$	7;20	$4(16_v;1,2)$
37	$SO_{10} \cdot SU_3 \cdot SU_7' \cdot SU_2'$	13;20	$4(10_v,1;1,2)+8(1,3;1,2)+4(1,\overline{3};1,2)$
38	$SU_7 \cdot SO_{12}'$	21;12	$12(7;1)+8(\overline{7};1)+24(1;1)$
39	$SU_7 \cdot SO_{10}' \cdot SU_2'^2$	21;14	$4(7;1,1,2)+8(\overline{7};1,1,2)+8(1;1,1,2)$
40	$SO_{14} \cdot SU_7'$	8;22	$12(14_v;1)+20(1;1)$
41	$SU_7 \cdot SO_{14}'$	22;9	$8(21;1)+4(\overline{21};1)+4(7;1)+4(\overline{7};1)+44(1;1)$
42	$SO_8 \cdot SU_4 \cdot SU_7'$	16;22	$12(8_s,1;1)+12(1,4;1)+8(1,\overline{4};1)$
43	$SU_7 \cdot SU_7'$	21;22	$12(7;1)+8(\overline{7};1)+24(1;1)$
44	$G_{6,3,2} \cdot E_7'$	23;3	$4(20,1,1;1)+8(6,3,1;1)+4(\overline{6},\overline{3},1;1)+20(1,1,2;1)$
45	$SO_{12} \cdot SU_2 \cdot G_{6,3,2}'$	10;23	$4(12_v,1;1,1,2)+12(1,2;1,1,2)$
46	$G_{6,3,2} \cdot SU_9'$	23;17	$4(20,1,1;1)+8(6,3,1;1)+4(\overline{6},\overline{3},1;1)+20(1,1,2;1)$
47	$SU_8 \cdot G_{6,3,2}'$	18;23	$4(8;1,1,2)+8(\overline{8};1,1,2)$
48	$SU_8 \cdot G_{6,3,2}'$	19;23	$8(8;1,1,2)+4(\overline{8};1,1,2)$
49	$SU_6 \cdot SU_3 \cdot E_8'$	24;0	$4(20,1;1)+4(6,3;1)+8(\overline{6},\overline{3};1)+44(1,1;1)$
50	$SU_6 \cdot SU_3 \cdot E_6' \cdot SU_3'$	24;4	$4(20,1;1,1)+4(6,3;1,1)+8(\overline{6},\overline{3};1,1)+44(1,1;1,1)$
51	$SO_{16} \cdot SU_6' \cdot SU_3'$	7;24	$12(16_v;1,1)$
52	$SO_{10} \cdot SU_3 \cdot SU_6' \cdot SU_3'$	13;24	$12(10_v,1;1,1)+8(1,3;1,1)+12(1,\overline{3};1,1)$
53	$SU_6 \cdot SU_2^2 \cdot E_7'$	25;2	$12(6,1,1;1)+8(\overline{6},1,1;1)+12(1,2,2;1)$
54	$SU_6 \cdot SU_2^2 \cdot E_6'$	25;6	$12(6,1,1;1)+8(\overline{6},1,1;1)+12(1,2,2;1)$
55	$SU_6 \cdot SU_2^2 \cdot SO_{12}' \cdot SU_2'$	25;11	$4(6,1,1;1,2)+8(\overline{6},1,1;1,2)+4(1,2,2;1,2)$
56	$SU_6 \cdot SU_2^2 \cdot SU_7' \cdot SU_2'$	25;20	$4(6,1,1;1,2)+8(\overline{6},1,1;1,2)+4(1,2,2;1,2)$
57	$SU_6 \cdot SU_2^2 \cdot SU_6' \cdot SU_3'$	25;24	$12(6,1,1;1,1)+8(\overline{6},1,1;1,1)+12(1,2,2;1,1)$
58	$SU_5 \cdot SU_4 \cdot E_7' \cdot SU_2'$	26;1	$8(5,1;1,2)+4(\overline{5},1;1,2)+4(1,6;1,2)$
59	$SU_5 \cdot SU_4 \cdot E_6' \cdot SU_2'$	26;5	$8(5,1;1,2)+4(\overline{5},1;1,2)+4(1,6;1,2)$
60	$SU_5 \cdot SU_4 \cdot SO_{10}' \cdot SU_2'$	26;15	$8(5,1;1,1)+12(\overline{5},1;1,1)+12(1,6;1,1)$
61	$SU_5 \cdot SU_4 \cdot G_{6,3,2}'$	26;23	$8(5,1;1,1,2)+4(\overline{5},1;1,1,2)+4(1,6;1,1,2)$